Solutions

1.1. The kinetic energy of the Boeing = 8×10^9 J. The mass of a mosquito is, say, 1 mg. The mosquito-antimosquito annihilation produces the energy $2\times10^{-6}\,(3\times10^8)^2=2\times10^{11}$ J.

1.2. $s=(3E)^2-0=9E^2=9(p^2+m^2)=88.9$ GeV^2; $m=\sqrt{s}=9.43$ GeV.

1.3. $\Gamma_{\pi^\pm}=h/\tau_{\pi^\pm}=\left(6.6\times10^{-16}\text{ eV s}\right)/\left(2.6\times10^{-8}\text{ s}\right)=25\text{ neV}$, $\Gamma_K=54$ neV, $\Gamma_{\Lambda}=2.5\,\mu$eV

1.4. $\tau_\rho=h/\Gamma_\rho=\left(6.6\times10^{-16}\text{ eV s}\right)/\left(1.49\times10^{12}\text{ eV}\right)=4.4\times10^{-24}\text{ s}$, $\tau_\omega=8\times10^{-23}$ s; $\tau_\phi=1.6\times10^{-22}$ s; $\tau_{K^*}=1.3\times10^{-23}$ s; $\tau_{J/\psi}=7\times10^{-21}$ s; $\tau_{\Delta}=5.5\times10^{-24}$ s.

1.5. Neglecting the recoil, the momentum transfer would be $q=E_e\sin\theta=2.1$ GeV, corresponding to the resolving power $D\approx197$ (MeV fm)/2100 (MeV) = 0.1 fm.

1.6. Our reaction is $p+p\rightarrow p+p+m$. In the CM frame the total momentum is zero. The lowest energy configuration of the system is when all particles in the final state are at rest.

a. Let us write down the equality between the expressions of s in the CM and L frames, i. e.

$$s=\left(E_p+m_p\right)^2-p_p^2=\left(2m_p+m\right)^2.$$

Recalling that $E_p^2=m_p^2+p_p^2$, we have $E_p=\dfrac{\left(2m_p+m\right)^2-2m_p^2}{2m_p}=m_p+2m+\dfrac{m^2}{2m_p}$.

b. The two momenta are equal and opposite because the two particles have the same mass, hence we are in the CM frame. The threshold energy E_p^* is given by $s=\left(2E_p^*\right)^2=\left(2m_p+m\right)^2$ which gives $E_p^*=m_p+m/2$.

c. $E_p=1.218$ GeV; $p_p=0.78$ GeV; $T_p=280$ MeV; $E_p^*=1.007$ GeV; $p_p^*=0.36$ GeV.

1.7. a. $s=\left(E_\gamma+m_p\right)^2-p_\gamma^2=\left(E_\gamma+m_p\right)^2-E_\gamma^2=\left(m_p+m_\pi\right)^2=1.16\text{ GeV}^2$, hence we have $E_\gamma=149$ MeV

b. $s=\left(E_\gamma+E_p\right)^2-\left(\mathbf{p}_\gamma+\mathbf{p}_p\right)^2=m_p^2+2E_\gamma E_p-2\mathbf{p}_\gamma\cdot\mathbf{p}_p$. For a given proton energy, s reaches a maximum for a head-on collision. Consequently, $\mathbf{p}_\gamma\cdot\mathbf{p}_p=-E_\gamma p_p$ and, taking into account that the energies are very large, $s=m_p^2+2E_\gamma\left(E_p+p_p\right)\approx m_p^2+4E_\gamma E_p$. In conclusion

$$E_p=\frac{s-m_p^2}{4E_\gamma}=\frac{\left(1.16-0.88\right)\times10^{18}\text{ eV}^2}{4\times10^{-3}\text{ eV}}=7\times10^{19}\text{ eV}=70\text{ EeV}.$$

c. The attenuation length is $\lambda=1/\left(\sigma\rho\right)=1.5\times10^{22}\text{ m}=5\text{ Mpc}\ \left(1\text{ Mpc}=3.1\times10^{22}\text{ m}\right)$

This is a short distance on the cosmological scale. The cosmic ray spectrum (Fig. 1.10) should not go beyond the above computed energy. This is called the Greizen, Zatzepin and Kusmin (GZK) bound. The AUGER observatory is now exploring this extreme energy region.

1.8. We call E_i the incident gamma energy and E_f the background gamma energy. At threshold $s=(2m_e)^2$.

For a given E_i, s is a maximum for head-on collisions: $s=\left(E_i+E_f\right)^2-\left(E_i-E_f\right)^2=4E_iE_f$.

Hence at threshold: $E_i = m_e^2 / E_f$.

a. $E_f = \frac{1}{\lambda} = \left(10^6 \text{ m}^{-1}\right) \times \left(1.97 \times 10^{-7} \text{ eV/m}^{-1}\right) \simeq 0.2 \text{ eV}$ and $E_i = \frac{\left(5 \times 10^5\right)^2 \text{ eV}^2}{0.2 \text{ eV}} = 1.25 \text{ TeV}$.

b. $E_i = \frac{\left(5 \times 10^5\right)^2 \text{ eV}^2}{10^{-3} \text{ eV}} = 250 \text{ TeV}$.

1.9. $s = \left(E_p + m_p\right)^2 - p_p^2 = \left(4m_p\right)^2 \Rightarrow E_{p,\min} = 7m_p = 6.6 \text{ GeV}$.

1.10. Calling E_b the beam energy at fixed target and E_p the energies of the colliding beams, the condition is $2m_pE_b = 4E_p^2$, hence he have $E_b = 100 \text{ PeV}$. This value is well above the ‘knee' of the cosmic ray spectrum, but it is much smaller than the GZK bound

1.11. We must consider the reaction

$$M \to m_1 + m_2 .$$

The figure defines the CM variables

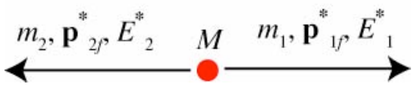

Fig. S.1

We can use equations (P1.5) and (P1.6) with $\sqrt{s}=M$, obtaining

$$E_{2f}^* = \frac{M^2 + m_2^2 - m_1^2}{2M};\ E_{1f}^* = \frac{M^2 + m_1^2 - m_2^2}{2M} .$$

The corresponding momenta are

$$\mathbf{p}_f^* \equiv \mathbf{p}_{1f}^* = -\mathbf{p}_{2f}^* = \sqrt{E_{1f}^{*2} - m_1^2} = \sqrt{E_{2f}^{*2} - m_2^2} .$$

1.12. In the Λ decay we have

$$E_\pi^* = \frac{m_\Lambda^2 - m_p^2 + m_\pi^2}{2m_\Lambda} = 0.17 \text{ GeV};\quad E_p^* = m_\Lambda - E_\pi^* = 0.94 \text{ GeV};\quad p^* = \sqrt{E_\pi^{*2} - m_\pi^2} = 0.1 \text{ GeV} .$$

And in the Ξ decay we have: $E_\pi^* = 0.20 \text{ GeV}$; $E_\Lambda^* = 1.12 \text{ GeV}$; $p^* = 0.14 \text{ GeV}$.

1.13. The expressions found in problem 1.11 become $E_2^* = \frac{M^2 - m_1^2}{2M}$ and $E_1^* = \frac{M^2 + m_1^2}{2M}$.

Since m_2=0, the CM momentum is $p^* = E_2^* = \frac{M^2 - m_1^2}{2M}$.

1.14. Let call x a coordinate along the beam. The velocity of the pions in L should not be larger than the velocity of the muon in the CM, i. e. $\beta_\pi \le \beta^*_{\mu\, x} \le \beta^*_\mu$. Let us use the formulae found in problem 1.14 to calculate the Lorentz parameters for the CM-L transformation

$$\beta_\mu^* = \frac{p^*}{E_\mu^*} = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2};\quad \gamma^*_\mu = \frac{E_\mu^*}{m_\mu} = \frac{m_\pi^2 + m_\mu^2}{2m_\mu m_\pi} \quad \Rightarrow \beta_\mu^*\gamma^*_\mu = \frac{m_\pi^2 - m_\mu^2}{2m_\mu m_\pi} .$$

The condition $\beta_\pi < \beta_\mu^*$ gives $p_\pi = \beta_\pi\gamma_\pi m_\pi < \beta_\mu^*\gamma^*_\pi m_\pi = \frac{m_\pi^2 - m_\mu^2}{2m_\mu} = 39.35 \text{ MeV}$.

1.15. When dealing with a Lorentz transformation problem, the first step is the accurate drawing of the momenta in the two frames and the definition of the kinematic variables.

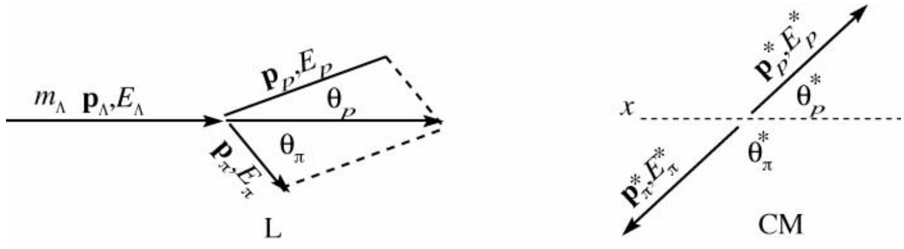

Fig. S.2

Using the expressions we found in the introduction we have:

a. $E_\pi^* = \dfrac{m_\Lambda^2 - m_p^2 + m_\pi^2}{2m_\Lambda} = 0.17$ GeV; $E_p^* = 0.95$ GeV; $p_\pi^* = p_p^* = \sqrt{E_\pi^{*2} - m_\pi^2} = 0.096$ GeV.

b. We calculate the Lorentz factors for the transformation:

$E_\Lambda = \sqrt{p_\Lambda^2 + m_\Lambda^2} = 2.29$ GeV; $\beta_\Lambda = \dfrac{p_\Lambda}{E_\Lambda} = 0.87$; $\qquad \gamma_\Lambda = \dfrac{E_\Lambda}{m_\Lambda} = 2.05$.

c. We do the transformation and calculate the requested quantities

$p_\pi \sin\theta_\pi = p_\pi^* \sin\theta_\pi^* = 0.096 \times \sin 210° = -0.048$ GeV

$p_\pi \cos\theta_\pi = \gamma_\Lambda \left(p_\pi^* \cos\theta_\pi^* + \beta_\Lambda E_\pi^* \right) = 2.05(0.096 \times \cos 210° + 0.87 \times 0.17) = 0.133$ GeV.

$\tan\theta_\pi = \dfrac{-0.048}{0.133} = -0.36 \qquad \theta_\pi = -20°; \qquad p_\pi = \sqrt{\left(p_\pi \sin\theta_\pi\right)^2 + \left(p_\pi \cos\theta_\pi\right)^2} = 0.141$ GeV.

$p_p \sin\theta_p = p_p^* \sin\theta_p^* = 0.048$ GeV

$p_p \cos\theta_p = \gamma_\Lambda \left(p_p^* \cos\theta_p^* + \beta_\Lambda E_p^* \right) = 2.05(0.096 \times \cos 30° + 0.87 \times 0.95) = 1.86$ GeV

$\tan\theta_p = \dfrac{0.048}{1.86} = 0.026 \qquad \theta_p = 1.5°$.

$p_p = \sqrt{\left(p_p \sin\theta_p\right)^2 + \left(p_p \cos\theta_p\right)^2} = 1.9$ GeV; $\theta = \theta_p - \theta_\pi = 21.5°$.

1.16. Remember to start by drawing the momentum vectors in the two reference frames, as in problem 1.15. We now have, being in non-relativistic conditions,

$$E_1 = E_3 + E_4 \quad \Rightarrow \quad \frac{p_1^2}{2m} = \frac{p_3^2}{2m} + \frac{p_4^2}{2m} \quad \Rightarrow \quad p_1^2 = p_3^2 + p_4^2 .$$

$$\mathbf{p}_1 = \mathbf{p}_3 + \mathbf{p}_4 \quad \Rightarrow \quad p_1^2 = p_3^2 + p_4^2 + 2\mathbf{p}_3 \cdot \mathbf{p}_4 = p_3^2 + p_4^2 \quad \Rightarrow \quad \mathbf{p}_3 \cdot \mathbf{p}_4 = 0 .$$

$\theta_{34} = \theta_{13} + \theta_{14} = \pi/2$: at non-relativistic speeds the angle between the final directions is 90°.

1.17. We continue to refer to the figure of problem 1.15. We shall solve our problem in two ways: by performing a Lorentz transformation and by using the Lorentz invariants.

We start with the first method. We calculate the Lorentz factors. The energy of the incident proton is $E_1 = \sqrt{p_1^2 + m_p^2} = 3.143$ GeV. Firstly, let us calculate the CM energy squared of the two-proton system (i. e. its mass squared).

$p_{pp} = p_1 = 3$ GeV; $E_{pp} = E_1 + m_p = 4.081$ GeV. Hence $s = 2m_p^2 + 2E_1 m_p = 7.656$ GeV^2.

The Lorentz factors are $\beta_{pp} = p_{pp}/E_{pp} = 0.735$ and $\gamma_{pp} = E_{pp}/\sqrt{s_{pp}} = 1.47$.

Since all the particles are equal, we have

$$E_1^* = E_2^* = E_3^* = E_4^* = \frac{\sqrt{s}}{2} = 1.385 \ \text{GeV}; \ p_1^* = p_2^* = p_3^* = p_4^* = \sqrt{E_1^{*2} - m_p^2} = 1.019 \ \text{GeV}.$$

We now perform the transformation. To calculate the angle we must calculate firstly the components of the momenta

$$p_3 \sin\theta_{13} = p_3^* \sin\theta_{13}^* = 1.019 \times \sin 10° = 0.177 \ \text{GeV}.$$

$$p_3 \cos\theta_{13} = \gamma\left(p_3^* \cos\theta_{13}^* + \beta E_3^*\right) = 1.473 \times (1.019 \times \cos 10° + 0.735 \times 1.385) = 2.978 \ \text{GeV}.$$

$$\tan\theta_{13} = \frac{0.177}{2.978} = 0.0594; \qquad \theta_{13} = 3°.$$

$$-p_4 \sin\theta_{14} = -p_4^* \sin\theta_{14}^* = -1.019 \times \sin 170° = -0.1769 \ \text{GeV}.$$

$$p_4 \cos\theta_{14} = \gamma\left(p_4^* \cos\theta_{14}^* + \beta E_4^*\right) = 1.473 \times (1.019 \times \cos 170° + 0.735 \times 1.385) = 0.0213 \ \text{GeV}.$$

$$\tan\theta_{14} = -0.1769 / 0.0213 = -8.305 \qquad \theta_{14} = -83° \qquad \Rightarrow \qquad \theta_{34} = \theta_{13} - \theta_{14} = 86°.$$

In relativistic conditions the angle between the final momenta in a collision between two equal particles is always, as in this example, smaller than 90°.

We now solve the problem using the invariants and the expressions in the introduction. We want the angle between the final particles in L. We then write down the expression of s in L in the initial state, which have already calculated, i. e.

$$s = (E_3 + E_4)^2 - (\mathbf{p}_3 + \mathbf{p}_4)^2 = m_3^2 + m_4^2 + 2E_3E_4 - 2\mathbf{p}_3 \cdot \mathbf{p}_4$$

that gives $\mathbf{p}_3 \cdot \mathbf{p}_4 = m_p^2 + E_3E_4 - s/2$ and hence $\cos\theta_{34} = \dfrac{m_p^2 + E_3E_4 - s/2}{p_3 p_4}$.

We need E_3 and E_4 (and their momenta); we can use (P.1.13) if we have t. With the data of the problem we can calculate t in the CM:

$$t = 2m_p^2 + 2p_i^{*2} \cos\theta_{13}^* - 2E_i^{*2} = 2p_i^{*2}\left(\cos\theta_{13}^* - 1\right) = 2 \times 1.019^2 \left(\cos 10° - 1\right) = -0.0316 \ \text{GeV}^2.$$

We then obtain

$$E_3 = \frac{s + t - 2m_p^2}{2m_p} = \frac{7.656 - 0.0316 - 2 \times 0.938^2}{2 \times 0.938} = 3.126 \ \text{GeV}; \qquad p_3 = 2.982 \ \text{GeV}.$$

From energy conservation we have

$$E_4 = E_1 + m_p - E_3 = 3.143 + 0.938 - 3.126 = 0.955 \ \text{GeV}; \qquad p_4 = 0.179 \ \text{GeV}.$$

Finally we obtain

$$\cos\theta_{34} = \frac{0.938^2 + 3.126 \times 0.955 - 7.656/2}{2.982 \times 0.179} = 0.0696 \qquad \Rightarrow \theta_{34} = 86°.$$

1.18. We must take into account that β_D is close to 1. We write

$$\gamma_D = \frac{E}{m_D} = 16.1; \quad \beta_D = \sqrt{\frac{\gamma_D^2 - 1}{\gamma_D^2}} = \sqrt{1 - \gamma_D^{-2}} \approx 1 - \frac{\gamma_D^{-2}}{2} = 0.998.$$

In the L reference frame the D life was $t = \dfrac{d}{\beta c} = 10$ ps long. In its rest-frame was $t_0 = t/\gamma_D = 0.62$ ps.

From $p_K = p_\pi = p^*$; $E_K + E_\pi = m_D$; $m_D = \sqrt{p_K^2 + m_K^2} + \sqrt{p_\pi^2 + m_\pi^2}$ we obtain

$$p_\pi = \sqrt{\left(\frac{m_D^2 + m_\pi^2 - m_K^2}{2m_D}\right)^2 - m_\pi^2} = 860 \text{ MeV}.$$

1.19. The distance travelled by a pion in a lifetime in the L frame is $l_0 = \gamma\beta c\tau_\pi$. If the initial number of pions is N_0, their number at the distance l is $N(l) = N_0 \exp\left(-\frac{l}{\gamma\tau_\pi\beta c}\right)$. Hence

$$\gamma\beta = \frac{l}{\tau_\pi c \ln\frac{N_0}{N(l)}} = \frac{20}{2.6\times10^{-8}\times3\times10^{-8}\times\ln(1/0.9)} = 24.3$$

and $p = m\gamma\beta = 0.14\times24.3 = 3.4$ GeV; $E = \sqrt{p^2 + m_\pi^2} = \sqrt{3.4^2 + 0.14^2} = 3.42$ GeV.

1.20. In this case the reference frames L and CM coincide. We have

$\mathbf{p}_{\pi^0} + \mathbf{p}_n = 0 \quad \Rightarrow \quad p_{\pi^0} = p_n = p^*$.

The total energy is $E = E_{\pi^0} + E_n = m_{\pi^-} + m_n = 1079$ MeV.

Subtracting the members of the two relationships $E_n^2 = p^{*2} + m_n^2$ and $E_{\pi^0}^2 = p^{*2} + m_{\pi^0}^2$ we obtain

$E_n^2 - E_{\pi^0}^2 = m_n^2 - m_{\pi^0}^2$

From $E_n = E - E_{\pi^\circ}$ we have $E_n^2 = E^2 + E_{\pi^0}^2 - 2EE_{\pi^0}$; and finally

$$E_{\pi^0} = \frac{E^2 + E_{\pi^0}^2 - E_n^2}{2E} = \frac{E^2 + m_{\pi^0}^2 - m_n^2}{2E} = 138.8 \text{ MeV}; T_n = E - E_{\pi^0} - m_n = 0.6 \text{ MeV}.$$

The Lorentz factors are $\gamma_{\pi^0} = E_{\pi^0}/m_{\pi^0} = 1.028$ nd $\beta_{\pi^0} = \sqrt{1 - 1/\gamma_{\pi^0}^2} = 0.23$.

The distance travelled in a lifetime is then

$l = \gamma_{\pi^0}\tau_{\pi^0}\beta_{\pi^0}c = 1.028\times8.4\times10^{-17}\times0.23\times3\times10^8 = 6$ nm.

1.21. The maximum momentum transfer is at background scattering. Eq. (6.25) gives in these conditions Q^2=4*EE'*, where *E'* is the energy of the scattered electron. Using Eq. (6.11) we have

$$Q_{\max}^2 = \frac{4E^2M}{M+2E} = \frac{4\times4\times56}{56+4} = 15 \text{ GeV}^2.$$

1.22. Having the α particle charge z=2, the cross section is

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2}{4E_k^2\sin^4\frac{\theta}{2}} = \frac{Z^2\alpha^2}{E_k^2}\frac{1}{(1-\cos\theta)^2}.$$

Integrating on the angles we have

$$\int_0^{2\pi} d\phi \int_{\theta_1}^{\theta_2} d\cos\theta \frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2}{E^2}2\pi\int_{\theta_1}^{\theta_2}\frac{1}{(1-\cos\theta)^2}d\cos\theta = \left(\frac{Z^2\alpha^2}{E^2}2\pi\right)\begin{matrix}\theta_2\\ \theta_1\end{matrix}\left|\frac{1}{\cos\theta-1}\right..$$

Hence $\left(\frac{d\sigma}{d\Omega}\right)_{\theta>90^0} / \left(\frac{d\sigma}{d\Omega}\right)_{\theta>10^0} = 0.0074$.

1.23. The requested rate is given by $R_s = \frac{\sigma(\theta>0.1)R_i t\rho N_A}{197\times(10^{-3}\text{kg})}$. We calculate the cross section

$$\sigma(\theta>\theta_1)=\int_{\theta_1}^{\pi} d\cos\theta\frac{d\sigma}{d\cos\theta}=\left(\frac{Z^2\alpha^2}{E_k^2}2\pi\right)\left(\frac{1}{\cos\pi-1}-\frac{1}{\cos\theta_1-1}\right)=\left(\frac{Z^2\alpha^2}{E_k^2}2\pi\right)\left(\frac{1}{1-\cos\theta_1}-\frac{1}{2}\right)$$

$=4.5\times10^3$ barn. The requested rate is

$$R_s=\frac{4.5\times10^{-25}(\mathrm{m}^2)\times10^3(\mathrm{s}^{-1})\times10^{-6}(\mathrm{m})\times1.93\times10^4(\mathrm{kg\ m}^{-3})\times6.02\times10^{23}}{197\times\left(10^{-3}\mathrm{kg}\right)}=26\ \mathrm{s}^{-1}.$$

1.24. At any angle the scattered electron energy reaches its maximum if the scattering is elastic and we have $E'=\dfrac{E}{1+\dfrac{E}{M}(1-\cos\theta)}=\dfrac{10}{1+\dfrac{10}{1}(1-0.87)}=4.3\ \mathrm{GeV}$.

1.25. $\cos\theta=1-\dfrac{E/E'-1}{E/M}=1-\dfrac{2.5-1}{20}=0.925 \qquad \theta=22^\circ$.

1.26. 0.5.

1.27. The equation of motion is $q\mathbf{v}\times\mathbf{B}=\dfrac{d\mathbf{p}}{dt}$. Since in this case the Lorentz factor γ is constant, we can write $q\mathbf{v}\times\mathbf{B}=\gamma m\dfrac{d\mathbf{v}}{dt}$. The centripetal acceleration is then: $\left|\dfrac{d\mathbf{v}}{dt}\right|=\dfrac{qvB}{\gamma m}=\dfrac{v^2}{\rho}$. Simplifying we obtain $p=qB\rho$. We now want pc in GeV, B in tesla and ρ in metres. Starting from $pc=qcB\rho$ we have

$$pc[\mathrm{GeV}]\times1.6\times10^{-10}[\mathrm{J/GeV}]=1.6\times10^{-19}[\mathrm{C}]\times3\times10^8[\mathrm{m/s}]\times B[\mathrm{T}]\times\rho[\mathrm{m}].$$

Finally in N.U.: $p[\mathrm{GeV}]=0.3\times B[\mathrm{T}]\times\rho[\mathrm{m}]$.

1.28. The number of protons in the unit volume of the target is $n_p=\dfrac{\rho\times N_A}{1\times10^{-3}}=3.6\times10^{28}\ \mathrm{m}^{-3}$; N_H and N_0 are linked by the relationship $N_H=N_o e^{-n_p\sigma l}$. Consequently, we have

$$\sigma=\frac{1}{n_p l}\ln\frac{N_o}{N_H}=\frac{10^{-29}}{0.36}\ln\frac{7.5}{6.9}=23.2\ \mathrm{mb}.$$

The statistical uncertainty about the incoming particles number is $\Delta N_i=\sqrt{N_i}$ and similarly for the outgoing number. The statistical error on the cross section is

$$\Delta\sigma=\left[\left(\frac{\partial\sigma}{\partial N_o}\right)^2 N_o+\left(\frac{\partial\sigma}{\partial N_H}\right)^2 N_H\right]^{\frac{1}{2}}=\frac{1}{n_p l}\left[\frac{1}{N_o}+\frac{1}{N_H}\right]^{\frac{1}{2}}=0.6\ \mathrm{mb}.$$

The final result is $\sigma=23.2\pm0.6$ mb.

1.29. The Lorentz factor of the antiproton is $\gamma=\sqrt{p^2+m^2}/m=1.62$ and its velocity $\beta=\sqrt{1-\gamma^{-2}}=0.787$. The condition in order to have the antiproton above the Cherenkov threshold is that the index is $n\geq1/\beta=1.27$.

If the index is n=1.5, the Cherenkov angle is given by $\cos\theta=1/n\beta=0.85$. Hence $\theta=32^\circ$.

1.30. The speed of a particle of momentum $p=m\gamma\beta$ is $\beta = \left(1+\frac{m^2}{p^2}\right)^{-1/2} \approx 1-\frac{m^2}{2p^2}$, that is a good approximation for speeds close to c. The difference between the flight times is $\Delta t = L\frac{m_2^2 - m_1^2}{2p^2}$ in N.U. In order to have Δt>600 ps, we need a base-length L>26 m.

1.31. The threshold condition is $n>\beta^{-1}$. Consequently, the index must satisfy the condition $1-\beta_\pi < n-1 < 1-\beta_K$. Since the speeds are very near to 1, we calculate the differences $1-\beta$ directly. From $\beta^{-1} = \frac{E}{p} \simeq 1+\frac{m^2}{2p^2}$ we have $\beta - 1 \simeq \frac{m^2}{2p^2}$. Hence $1-\beta_\pi = 2.45\times 10^{-5}$ and $1-\beta_K = 3.05\times 10^{-4}$. Consequently the condition on the pressure is 8.2 kPa<Π<102 kPa.

1.32. Superman saw the light blue shifted due to Doppler effect. Taking for the wavelengths λ_R=650 nm and λ_G=520 nm, we have $\nu_G/\nu_R = 1.25$. Solving for β the Doppler shift expression $\nu_G = \nu_R\sqrt{\frac{1+\beta}{1-\beta}}$, we obtain β=0.22.

1.33

1. The minimum velocity is $\beta_{\min} = \frac{1}{n} = 0.75$; 2.The minimum kinetic energy for a proton is $E_{kin,\min}(p) = m_p\left(\frac{1}{\sqrt{1-\beta_{\min}^2}} - 1\right) = 938\times 0.51 = 480\ \text{MeV}$ and of the pion: $E_{kin,\min}(\pi) = m_\pi\left(\frac{1}{\sqrt{1-\beta_{\min}^2}} - 1\right) = 139.6\times 0.51 = 71.2\ \text{MeV}$; 3. the Lorentz factor is $\gamma = \frac{E_\pi}{m_\pi} = \frac{400}{139.6} = 2.87$ and $\beta = \sqrt{1-\gamma^{-2}} = 0.94$. The Cherenkov angle is then $\theta = \cos^{-1}\left(\frac{1}{\beta n}\right) = 36.9^0$

1.34.

a. The Cherenkov threshold is $\beta_{thr}^{-1} = n$. For a generic mass m

$$\beta^{-1} - 1 = \frac{E}{p} - 1 = \sqrt{\frac{p^2+m^2}{p^2}} - 1 \simeq \frac{1}{2}\frac{m^2}{p^2}$$

Threshold condition for pions is given by

$$\beta^{-1} - 1 = n - 1 = 3\times 10^{-9}\Pi = 3\times 10^{-9}\times 5.2\times 10^3 = 1.56\times 10^{-5}$$

and $p = \frac{m_\pi}{\sqrt{2\times 1.56\times 10^{-5}}} = 25\ \text{GeV}$

b. $\Pi(K) = 5.2\times 10^3\ \frac{m_K^2}{m_\pi^2}$=$5.2\times 10^3\ \left(\frac{0.494}{0.140}\right)^2$=$6.5\times 10^4$ Pa=650 mbar

c. $\Pi(p) = 5.2\times 10^3\ \frac{m_p^2}{m_\pi^2}$=$5.2\times 10^3\ \left(\frac{0.938}{0.140}\right)^2$=$2.33\times 10^5$ Pa=2330 mbar

1.35.

1. $E = p = 0.3 \times B \times R = 0.3 \times 10^{-9} \times 10^{13} = 3$ TeV
2. $E = p = 0.3 \times B \times R = 0.3 \times 5 \times 10^{-11} \times 3 \times 10^{20} = 5 \times 10^{9}$ GeV

1.36

1. The total energy of the deuterons is $E_d = m_d + T_d = 1875.7$ MeV. The motion of the deuterons is not relativistic. Their momentum is
$p_d = \sqrt{2m_d T_d} = \sqrt{2 \times 1875.6 \times 0.13} = 61.25$ MeV. This is also the total momentum, which is so small that in this case the L frame is also in practice the CM frame.
The CM energy squared is $\sqrt{s} = \sqrt{\left(E_d + m_t\right)^2 - p_d^2} \simeq E_d + m_t = 4684.6$. The result could be obtained by simply summing the two masses and the deuteron kinetic energy. This because the situation is non relativistic. The total kinetic energy available after the reaction is $E_{kin,t} = E_d + m_t - m_\alpha - m_n = 17.6$ MeV, which is mainly taken by the lighter particle, the neutron. To be precise

$$T_n = \frac{s + m_n^2 - m_\alpha^2}{2\sqrt{s}} - m_n = \frac{4684.6^2 + 939.6^2 - 3727.4^2}{2 \times 4684.6} - 939.6 = 953.6 - 939.6 = 14.0 \text{ MeV}$$

and

$$T_\alpha = \frac{s + m_\alpha^2 - m_n^2}{2\sqrt{s}} - m_n = \frac{4684.6^2 + 3727.4^2 - 939.6^2}{2 \times 4684.6} - 3727.4 = 3.6 \text{ MeV}$$

2. The flux is $\Phi = \frac{I_n}{4\pi R^2} = \frac{3 \times 10^{10}}{4\pi \times 1^2} = 2.4 \times 10^{9}$ nutrons/(m^{2}s).

3. We can calculate the momentum of the neutron non relativistically
$p_n = \sqrt{2m_n T_n} = \sqrt{2 \times 939.6 \times 14} = 162.2$ MeV, and its velocity
$\beta_n = \frac{p_n}{E_n} = \frac{162.2}{953.6} = 0.17$ $\quad\quad \upsilon_n = 5.1 \times 10^{7}$ m/s. We need 1 ns time resolution

1.37. The minimum momentum to resolve the structure is $p_{\min} = \frac{197 \text{ MeV fm}}{R_A} = 50$ MeV. The momentum of the neutron of (non relativistic) kinetic energy $E_{k,0}$ is $p_n = \sqrt{2m_n E_{k,0}}$.

The coherence condition is $\sqrt{2m_n E_{k,0}} < p_{\min}$ or $E_{k,0} < \frac{p_{\min}^2}{8m_n} = \frac{50^2}{4 \times 940} = 0.7$ MeV.

Call p_0 the initial neutron momentum, corresponding to the kinetic energy E_{k0}, p_1 and E_{k1} the momentum and kinetic energy of the final neutron, p_2 and E_{k2} those of the recoiling Ar nucleus. Momentum and kinetic energy conservation in the non-relativistic kinematics of the elastic background scattering give

$$\frac{p_0^2}{2m_n} = \frac{p_1^2}{2m_n} + \frac{p_2^2}{2m_{Ar}}$$
$$p_0 = p_2 - p_1$$

From the first equation we have $p_0^2 = p_1^2 + \frac{m_n}{m_{Ar}} p_2^2$

And from the second $p_0^2 = p_1^2 + p_2^2 - 2p_1 p_2$

Equating the second members we obtain (provided $p_2 \neq 0$) $p_1 = \frac{1}{2} p_2 \left(1 - \frac{m_n}{m_{Ar}}\right)$.

We substitute this expression in the momentum conservation equation, obtaining $p_2 = \frac{2p_o}{1 + \frac{m_n}{m_{Ar}}} = \frac{2 \times 50}{1 + \frac{0.94}{37.2}} = \frac{100}{1.025} = 97.6 \text{ MeV}$.

The recoil kinetic energy is $E_{k2} = \frac{p_2^2}{2m_{Ar}} = \frac{97.6^2}{2 \times 37200} = 130 \text{ keV}$

1.38.

(a) $E_2 = \frac{h}{\lambda} = \frac{1240 \text{ eV nm}}{694 \text{ nm}} = 1.79 \text{ eV}$.

The CM energy for the head-on geometry is $s = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 = 2E_1E_2 + 2E_1E_2$.

At threshold $s = 4E_1E_2 = (2m_e)^2$, that is $E_1 = \frac{m_e^2}{E_2} = \frac{(0.5)^2}{1.79 \times 10^{-6}} = 140 \text{ GeV}$

(b) $1 - \beta = 1 - \frac{\mathbf{p}_1 + \mathbf{p}_2}{E_1 + E_2} = 1 - \frac{E_1 - E_2}{E_1 + E_2} = 1 - \frac{1 - E_2/E_1}{1 + E_2/E_1} \simeq 2\frac{E_2}{E_1} = 2.6 \times 10^{-11}$

(c) $s = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 = 2E_1E_2 - 2E_1E_2 = 0$. The mass is zero for any values of the two energies.

2.1. From the result of the Problem 11.13 we have $p^* = E_\nu^* = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} = 29.8 \text{ MeV}$. From this we obtain $E_\mu^* = m_\pi - p^* = 110 \text{ MeV}$.

2.2. a) $p_\nu^* = E_\nu^* = p_\mu^* = 236 \text{ MeV}$; $E_\mu^* = 259 \text{ MeV}$

b) p_K=5 GeV, hence $E_K = \sqrt{p_K^2 + m_K^2} = 5.02$ GeV; $\gamma = \frac{E_K}{m_K} = 10.2$; $\gamma\beta = \frac{p_K}{m_K} = 10.1$.

The muons with maximum energy in L are those that are emitted backwards by the kaon. Their momentum is $p_\mu = \gamma p_\mu^* + \beta\gamma E_\mu^* = 10.2 \times 0.236 + 10.1 \times 0.259 = 5.02 \text{ GeV}$.

2.3. The second gamma moves backwards. The total energy is $E = E_1 + E_2$; the total momentum is $P = p_1 - p_2 = E_1 - E_2$. The square of the mass of the two-gamma system is equal to the square of the pion mass: $m_{\pi^0}^2 = \left(E_1 + E_2\right)^2 - \left(E_1 - E_2\right)^2 = 4E_1E_2$, from which we obtain $E_2 = \frac{m_{\pi^0}^2}{4E_1} = \frac{135^2}{4\times 150} = 30.4 \text{ MeV}$. The speed of the π^0 is $\beta = \frac{P}{E} = \frac{E_1 - E_2}{E_1 + E_2} = 0.662$.

2.4. The Lorentz factor for E_μ=5 GeV is $\gamma = E_\mu / m_\mu = 47$. In its rest frame the distance of the Earth surface is $l_0 = l/\gamma = 630 \text{ m}$. For E_μ=5 TeV, the distance of the Earth is $l_0 = l/\gamma = 0.63 \text{ m}$. The first muon travels in a lifetime $\gamma\beta c\tau \simeq \gamma c\tau = 28 \text{ km}$, the second would travel 28 000 km if it did not hit the surface first.

2.5. The Lorentz factor for E_π=5 GeV is $\gamma = E_\pi / m_\pi = 36$. In its rest frame it sees the Earth's surface at the distance $l_0 = l/\gamma = 830 \text{ m}$. In a lifetime it travels $\gamma c\tau = 280 \text{ m}$. We see that only a few such pions survive. To find them we must go to high altitude.

2.6. The momenta of the electrons are $p = 0.3B\rho$=12 MeV. The gamma energy is E_γ=24 MeV.

2.8. Since the decay is isotropic, the probability of observing a photon is a constant $P\left(\cos\theta^*, \phi^*\right) = K$. We determine K by imposing that the probability of observing a photon at any angle is 2, i. e. the number of photons.

We have $2 = \int K \sin\theta^* d\theta^* d\phi = \int_0^{2\pi} d\phi \int_0^{\pi} K d(\cos\theta^*) = K 4\pi$. Hence $K = 1/2\pi$ and $P\left(\cos\theta^*, \phi^*\right) = 1/2\pi$.

The distribution is isotropic in azimuth in L too. To have the dependence of θ, that is given by $P\left(\cos\theta\right) \equiv \frac{dN}{d\cos\theta} = \frac{dN}{d\cos\theta^*}\frac{d\cos\theta^*}{d\cos\theta}$, we must calculate the 'Jacobian' $J = \frac{d\cos\theta^*}{d\cos\theta}$.

Calling β and γ the Lorentz factors of the transformation and taking into account that $p^* = E^*$, we have

$$p\cos\theta = \gamma\left(p^*\cos\theta^* + \beta E^*\right) = \gamma p^*\left(\cos\theta^* + \beta\right)$$

$$E = p = \gamma\left(E^* + \beta p^*\cos\theta^*\right) = \gamma p^*\left(1 + \beta\cos\theta^*\right).$$

We differentiate the first and third members of these relationships, taking into account that p^* is a constant. We obtain

$$dp \times \cos\theta + p \times d\left(\cos\theta\right) = \gamma p^* d\left(\cos\theta^*\right) \quad \Rightarrow \quad \frac{dp}{d\cos\theta^*}\cos\theta + p\frac{d\cos\theta}{d\cos\theta^*} = \gamma p^*.$$

$$dp = \gamma\beta p^* d\left(\cos\theta^*\right) \quad \Rightarrow \quad \frac{dp}{d\cos\theta^*} = \gamma\beta p^*$$

and $J^{-1} = \frac{d\cos\theta}{d\cos\theta^*} = \gamma\frac{p^*}{p}\left(1 - \beta\cos\theta\right)$.

The inverse transformation is $E^* = \gamma\left(E - \beta p\cos\theta\right)$, i. e. $p^* = \gamma p\left(1 - \beta\cos\theta\right)$, giving

$$J^{-1} = \frac{d\cos\theta}{d\cos\theta^*} = \gamma^2\left(1 - \beta\cos\theta\right)^2.$$

Finally we obtain $P\left(\cos\theta\right) \equiv \frac{dN}{d\cos\theta} = \frac{1}{2\pi}\gamma^{-2}\left(1 - \beta\cos\theta\right)^{-2}$

2.10. $\mu_e / \mu_\mu = m_\mu / m_e = 207$; $\mu_e / \mu_\tau = m_\tau / m_e = 3477$.

2.11. The energy needed to produce an antiproton is minimum when the Fermi motion is opposite to the beam direction. If E_f is the total energy of the target proton and p_f its momentum, the threshold condition is $(E_p+E_f)^2-(p_p-p_f)^2=(4m_p)^2$. From this we have $E_pE_f + p_pp_f = 7m_p^2$. We simplify by setting $p_p \simeq E_p$ obtaining

$$E_p = \frac{7m_p^2}{E_f + p_f} \simeq \frac{7m_p^2}{m_p + p_f} \simeq 7m_p\left(1 - \frac{p_f}{m_p}\right) = 5.5 \text{ GeV}.$$

This value should be compared to E_p=6.6 GeV on free protons.

2.12. By differentiating (1.79) we obtain $\Delta\theta = 0.3BL\Delta p / p^2$. The slit of opening d at the distance l defines the angle within $\Delta\theta = d/l$. The requested distance is then $l = \dfrac{d \times p}{0.3BL\Delta p / p} = 3.3 \text{ m}$.

2.13. Considering the beam energy and the event topology, the event is probably an associate production of a K^0 and a Λ. Consequently the V^0 may be one of these two particles. The negative track is in both cases a π, while the positive track may be a π or a proton. We need to measure the mass of the V. With the given data we start by calculating the Cartesian components of the momenta

$p_x^- = 121 \times \sin(-18.2°)\cos 15° = -36.5$ MeV; $p_y^- = 121 \times \sin(-18.2°)\sin 15° = -9.8$ MeV;

$p_z^- = 121 \times \cos(-18.2°) = 115$ MeV.

$p_x^+ = 1900 \times \sin(20.2°)\cos(-15°) = 633.7$ MeV; $p_y^- = 1900 \times \sin(20.2°)\sin(-15°) = -169.8$ MeV;

$p_z^- = 1900 \times \cos(20.2°) = 1783.1$ MeV.

Summing the components, we obtain the momentum of the V, i. e. $p = 1998$ MeV.

The energy of the negative pion is $E^- = \sqrt{(p^-)^2 + m_\pi^2} = 185$ MeV. If the positive track is a π its energy is $E_\pi^+ = \sqrt{(p^+)^2 + m_\pi^2} = 1905$ MeV, while if it is a proton its energy is $E_p^+ = 2119$ MeV. The energy of the V is $E_\pi^V = 2090$ MeV in the first case, $E_p^V = 2304$ MeV in the second case. The mass of the V is consequently $m_\pi^V = \sqrt{E_\pi^{V2} - p^2} = 620$ MeV in the first hypothesis, $m_p^V = 1150$ MeV in the second. Within the ±4% uncertainty, the first hypothesis is incompatible with any known particle, while the second is compatible with the particle being a Λ.

2.14.

1. The CM energy squared is $s = (E_\nu + m_n)^2 - p_\nu^2 = m_n^2 + 2m_nE_\nu$. The threshold condition is

$$s = (m_e + m_p)^2 = m_p^2 + m_e^2 + 2m_em_p.$$

Hence, the threshold condition is $E_\nu = \dfrac{(m_e + m_p)^2 - m_n^2}{2m_n} < 0$, meaning that there is no threshold, the reaction proceeds also at zero neutrino energy.

2. The threshold condition is $s=\left(m_\mu+m_p\right)^2=m_p^2+m_\mu^2+2m_\mu m_p$. The threshold energy is

$$E_\nu=\frac{\left(m_\mu+m_p\right)^2-m_n^2}{2m_n}=\frac{(105.7+938.3)^2-939.6^2}{2\times 939.6}=110\text{ MeV}$$

3. The threshold energy is

$$E_\nu=\frac{\left(m_\tau+m_p\right)^2-m_n^2}{2m_n}=\frac{(1777+938.3)^2-939.6^2}{2\times 939.6}=3.45\text{ GeV}.$$

2.15. We first find an expression valid in both cases. Call $E_{\gamma 1}$ and $p_{\gamma 1}= E_{\gamma 1}$ the energy and momentum of the initial photon and $E_{\gamma 2}$ and $p_{\gamma 2}= E_{\gamma 2}$ those of the final one. Similarly E_{e1}, p_{p1} and E_{e2}, p_{p2} for the electron.

The initial values of energies and momenta are given; hence the total energy and momentum and CM energy squared

$E_T=E_{e1}+E_{\gamma 1}$ $\quad p_T=p_{e1}-E_{\gamma 1}$ $\quad s=E_T^2-p_T^2$.

Energy conservation gives $E_T=E_{e2}+E_{\gamma 2}$ $\quad p_T=p_{e2}-E_{\gamma 2}$.

We can eliminate the final energy and momentum of the electron by imposing $E_{e2}^2-p_{e2}^2=m_e^2$.

$E_{e2}=E_T-E_{\gamma 2}$ $\quad p_{e2}=p_T+E_{\gamma 2}$. Hence: $\left(E_T-E_{\gamma 2}\right)^2-\left(p_T+E_{\gamma 2}\right)^2=m_e^2$. Solving for $E_{\gamma 2}$ we have $E_{\gamma 2}=\dfrac{s-m_e^2}{2\left(E_T+p_T\right)}$.

1. We have, in MeV: $E_{\gamma 1}=0.511$, E_{e1}=0.511, p_{e1}=0.

E_T=1.02, p_T=0.511, s=0.78 and $E_{\gamma 2}=\dfrac{s-m_e^2}{2\left(E_T+p_T\right)}=\dfrac{0.78-0.511^2}{2(1.02+0.511)}=0.170\text{ MeV}$.

2. We have $E_{\gamma 1}=0.511$, E_{e1}=1.02, $p_{e1}=\sqrt{1.02^2-0.511^2}=0.88$.

E_T=1.53, p_T=0.511, s=2.08 and $E_{\gamma 2}=\dfrac{s-m_e^2}{2\left(E_T+p_T\right)}=\dfrac{2.08-0.511^2}{2(1.53+0.511)}=0.446\text{ MeV}$.

2.16. The LASER photon energy is $E_{\gamma i}=\dfrac{h}{\lambda}=\dfrac{1240\text{ eV nm}}{694\text{ nm}}=1.79\text{ eV}$.

The electron initial momentum (we shall need its difference from energy) is

$$p_{ei}=\sqrt{E_{ei}^2-m_e^2}\simeq E_{ei}-\frac{m_e^2}{2E_{ei}}$$

The total energy and momentum are $E_T=E_{ei}+E_{\gamma i}$ $\quad p_T=p_{ei}-E_{\gamma i}$

Energy conservation gives $E_T=E_{ef}+E_{\gamma f}$ $\quad p_T=E_{\gamma f}-p_{ef}$

We can eliminate the final energy and momentum of the electron by imposing $E_{ef}^2-p_{ef}^2=m_e^2$.

$E_{ef}=E_T-E_{\gamma f}$ $\quad p_{ef}=E_{\gamma f}-p_T$. Hence: $\left(E_T-E_{\gamma f}\right)^2-\left(E_{\gamma f}-p_T\right)^2=m_e^2$. Solving for $E_{\gamma f}$ we have $E_{\gamma f}=\dfrac{s-m_e^2}{2\left(E_T-p_T\right)}$.

$$E_T - p_T = \left(E_{ei} + E_{\gamma i}\right) - \left(p_{ei} - E_{\gamma i}\right) \simeq \frac{m_e^2}{2E_{ei}} + 2E_{\gamma i} =$$

$$= \frac{0.5^2 \times 10^{-6}}{2 \times 20} + 2 \times 1.79 \times 10^{-9} = \left(6.25 + 3.58\right)10^{-9} \text{ GeV} = 9.83 \text{ eV}$$

$s = \left(E_{\gamma i} + E_{ei}\right)^2 - \left(E_{\gamma i} - p_{ei}\right)^2 = m_e^2 + 4E_{\gamma i}E_{ei}$. Hence

$s - m_e^2 = 4E_{\gamma i}E_{ei} = 4 \times 1.79 \times 20 \times 10^9 \text{ eV}^2 = 14.3 \times 10^{10} \text{ eV}^2$, and

$$E_{\gamma f} = \frac{s - m_e^2}{2\left(E_T - p_T\right)} = \frac{14.3 \times 10^{10}}{2 \times 9.83} = 7.3 \text{ GeV}$$

2.17. The kinetic energy is $T = \sqrt{p^2 + m^2} - m$

For a proton we have $T = \sqrt{23^2 + 938.3^2} - 938.3 = 280$ keV

For a positron we have $T = \sqrt{23^2 + 0.51^2} - 0.51 = 22.5$ MeV

2.18. $E \simeq p = 0.3BR = 0.3 \times 0.3 \times 0.14 = 12.6$ MeV.

2.19. In problem 2.1 we already calculated the CM momentum $p^* = \dfrac{m_\pi^2 - m_\mu^2}{2m_\pi} = 29.8$ MeV. The CM muon energy is $E_\mu^* = \sqrt{p^{*2} + m_\mu^2} = 110$ MeV. For the Lorentz transformation to the L frame we have $\beta \simeq 1$ and $\gamma = \dfrac{E_\pi}{m_\pi} = \dfrac{200}{0.14} == 1400$. The maximum and minimum muon energies are

$E_{\mu \min}^{\max} = \gamma\left(E_\mu^* \pm \beta p^*\right) = 1400\left(0.110 \pm 0.030\right) = 112 - 196$ GeV.

3.2. Strangeness conservation requires that a K^+ or a K^0 is produced together with the K^-. The third component of the isospin in the initial state is –1/2. Let us check if it is conserved in the two reactions. The answer is yes for $\pi^- + p \to K^- + K^+ + n$ because in the final state we have $I_z = -\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = +\frac{1}{2}$, and yes also for $\pi^- + p \to K^- + K^0 + p$ because in the final state we have $I_z = -\frac{1}{2} - \frac{1}{2} + \frac{1}{2} = -\frac{1}{2}$. The threshold of the first reaction is just a little smaller of that of the second reaction because $m_n + m_{K^+} < m_p + m_{K^0}$ (1433 MeV < 1436 MeV). For the former we have

$$E_\pi = \frac{\left(2m_K + m_n\right)^2 - m_\pi^2 - m_p^2}{2m_p} = 1.5 \text{ GeV}.$$

3.3. To conserve both strangeness and baryonic number a pair of $\Lambda\bar{\Lambda}$ must be produced. The reaction is $\pi^- + p \to \Lambda + \bar{\Lambda} + n$. On free protons, we obtain $E_\pi^0 = 4.9$ GeV. The threshold energy on bound protons, having Fermi momentum p_f, following Problem 2.11, is found to be $E_\pi = E_\pi^0\left(1 - p_f / m_p\right) = 4.1$ GeV.

In the first case pions in the beam have γ=35 and β≈1. The flux at the emulsion stack is N= 0.97 10^6 π/cm^2.

3.4. 1. OK, S; 2. OK, W; 3. Violates $\mathcal{L}_\mu$; 4. OK, EM; 5. Violates C; 6. Cannot conserve both energy and momentum; 7. violates $\mathcal{B}$ and S; 8. violates $\mathcal{B}$ and S; 9. violates J and $\mathcal{L}_e$; 10. violates energy conservation.

3.5. 1. Violates $\mathcal{L}_e$ and $\mathcal{L}_\mu$; 2. Violates charge conservation; 3. Violates $\mathcal{B}$, I and I_z; 4. Violates the charge; 5. Violates $\mathcal{B}$, I and I_z; 6. Violates charge and S; 7. Violates S and I_z; 8.Violates S and I_z; 9. Violates energy conservation; 10. Violates $\mathcal{L}_e$ and $\mathcal{L}$.

3.6. a) J and $\mathcal{L}$, b) $\mathcal{B}$ and $\mathcal{L}$, c) energy conservation ; d) electric charge.

3.7. a) $0+0\to -1+0$, NO; b) $0+0\to 1-1$, YES; c) $-1+0\to 1-2+0$, YES; d) $1+0\to -1-2+0$, NO; e) $-1+0\to -3+1+1$, YES.

3.8. a) NO for J and $\mathcal{L}$; b) NO for J and $\mathcal{L}$; c) YES; d) NO for $\mathcal{L}$; e) YES; f) NO for $\mathcal{L}_e$ and $\mathcal{L}_\mu$; g) NO for $\mathcal{L}$; h) YES.

3.9. $\left|\pi^- p\right\rangle = \left|1,-1\right\rangle\left|\frac{1}{2},+\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle$

$\left|\pi^+ p\right\rangle = \left|1,+1\right\rangle\left|\frac{1}{2},+\frac{1}{2}\right\rangle = \left|\frac{3}{2},+\frac{3}{2}\right\rangle$

$\left|\Sigma^0 K^0\right\rangle = \left|1,0\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle$

$\left|\Sigma^- K^+\right\rangle = \left|1,-1\right\rangle\left|\frac{1}{2},+\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle$

$\left|\Sigma^+ K^+\right\rangle = \left|1,+1\right\rangle\left|\frac{1}{2},+\frac{1}{2}\right\rangle = \left|\frac{3}{2},+\frac{3}{2}\right\rangle$

$\left\langle K^+\Sigma^+\middle|\pi^+ p\right\rangle = A_{3/2}$; $\left\langle \Sigma^- K^+\middle|\pi^- p\right\rangle = \sqrt{\frac{1}{3}}\sqrt{\frac{1}{3}}A_{3/2} = \frac{1}{3}A_{3/2}$; $\left\langle \Sigma^0 K^0\middle|\pi^- p\right\rangle = \sqrt{\frac{2}{3}}\sqrt{\frac{1}{3}}A_{3/2} = \frac{\sqrt{2}}{3}A_{3/2}$

Hence: $\sigma\left(\pi^+ p\to\Sigma^+ K^+\right):\sigma\left(\pi^- p\to\Sigma^- K^+\right):\sigma\left(\pi^- p\to\Sigma^0 K^0\right) = 9:1:2$

3.10. From the expressions found in the solution of problem 3.9, we have

$\sigma(1):\sigma(2):\sigma(3) = 2\left|A_{3/2}-A_{1/2}\right|^2 : \left|A_{3/2}+2A_{1/2}\right|^2 : 9\left|A_{3/2}\right|^2$

3.11. $\sigma(1)/\sigma(2) = 1$

3.12. We can proceed as in the previous solutions or also as follows.

$\left|p,d\right\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle\left|0,0\right\rangle = \left|\frac{1}{2},+\frac{1}{2}\right\rangle$

$\left|\frac{1}{2},+\frac{1}{2}\right\rangle = -\sqrt{\frac{1}{3}}\left|1,0\right\rangle\left|\frac{1}{2},+\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|1,1\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle = -\sqrt{\frac{1}{3}}\left|\pi^0\right\rangle\left|\mathrm{He}^3\right\rangle + \sqrt{\frac{2}{3}}\left|\pi^+\right\rangle\left|\mathrm{H}^3\right\rangle$

$\sigma\left(p+d\to \mathrm{He}^3+\pi^0\right)/\sigma\left(p+d\to \mathrm{H}^3+\pi^+\right) = 1/2$

3.13.From $\left|p,p\right\rangle = \left|\frac{1}{2},+\frac{1}{2}\right\rangle\left|\frac{1}{2},+\frac{1}{2}\right\rangle = \left|1,+1\right\rangle$ and $\left|d,\pi^+\right\rangle = \left|0,0\right\rangle\left|1,+1\right\rangle = \left|1,+1\right\rangle$ we have $\left\langle d,\pi^+\middle|p,p\right\rangle = A_1$.

From $|p,n\rangle = \left|\frac{1}{2},+\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}|0,0\rangle + \frac{1}{\sqrt{2}}|1,0\rangle$ and $\left|d,\pi^0\right\rangle = |0,0\rangle|1,0\rangle = |1,0\rangle$ we have $\left\langle d,\pi^0 \middle| p,n\right\rangle = \frac{1}{\sqrt{2}}A_1$. Finally we obtain $\sigma\left(pp \to d\pi^+\right)/\sigma\left(pn \to d\pi^0\right) = 2$.

3.14. $\left|K^-,\mathrm{He}^4\right\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle|0,0\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle$.

$$\left|\Sigma^0,\mathrm{H}^3\right\rangle = |1,0\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle \qquad \Rightarrow \left\langle K^-,\mathrm{He}^4 \middle| \Sigma^0,\mathrm{H}^3\right\rangle = \sqrt{\frac{1}{3}}A_{1/2}$$

$$\left|\Sigma^-,\mathrm{He}^3\right\rangle = |1,-1\rangle\left|\frac{1}{2},+\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle \qquad \Rightarrow \left\langle K^-,\mathrm{He}^4 \middle| \Sigma^-,\mathrm{He}^3\right\rangle = -\sqrt{\frac{2}{3}}A_{1/2}$$

$\sigma\left(K^- + \mathrm{He}^4 \to \Sigma^0\mathrm{H}^3\right)/\sigma\left(K^- + \mathrm{He}^4 \to \Sigma^-\mathrm{He}^3\right) = 1/2$.

3.15. $\sigma(1):\sigma(2):\sigma(3) = \left|-\frac{1}{\sqrt{6}}A_0 + \frac{1}{2}A_1\right|^2 : \left|\frac{1}{\sqrt{6}}A_0\right|^2 : \left|\frac{1}{\sqrt{6}}A_0 + \frac{1}{2}A_1\right|^2$.

3.16. $\sigma\left(\pi^- p \to \pi^- p\right)/\sigma\left(\pi^- p \to \pi^0 n\right) = \left|\frac{2}{3}A_{1/2} + \frac{1}{3}A_{3/2}\right|^2 / \left|-\frac{\sqrt{2}}{3}A_{1/2} + \frac{\sqrt{2}}{3}A_{3/2}\right|^2$.

3.17. a) The initial parity is $P_i = P(\pi)P(d)(-1)^{l_i} = (-)(+)(+) = -$ and the final one is $P_f = P(n)P(n)(-1)^{l_f} = (-1)^{l_f}$. Parity conservation requires l_f=1,3,5… Angular momentum conservation requires that $l_f<3$. Only $l_f=1$ remains. The two-neutron wave function must be completely antisymmetric. Since the spatial part is antisymmetric, the spin part must be symmetric. In conclusion the state is $^3\mathrm{S}_1$, with total spin S=1.

b) Since P_i=+, l_f is even. The spin function is antisymmetric. Hence the state is $^1\mathrm{S}_0$ and its total spin is S=0.

3.18. From (3.18) the initial charge conjugation is $C=(-1)^{l+s}$. The final one is $C(n\gamma)=(-1)^n$. The charge conjugation is conserved if $l+S+n$=even.

In the ortho-positronium $l+s=1$, consequently n= odd. The minimum number of photons is n=3 (n=1 forbidden by energy-momentum conservation).

In the para-positronium $l+s=0$, hence n= even and the minimum number of photons is n=2.

3.19.

1. $C(\bar{p}p)=(-1)^{l+s}=C(n\pi^0)=+$. Then $l+s$=even. The possible states are $^1\mathrm{S}_0$, $^3\mathrm{P}_1$, $^3\mathrm{P}_2$, $^3\mathrm{P}_3$, $^1\mathrm{D}_2$.

2. The orbital momentum is even, because the wave function of the $2\pi^0$ state must be symmetric. Since the total angular momentum is just orbital momentum, only the states $^1\mathrm{S}_0$, $^3\mathrm{P}_2$, $^1\mathrm{D}_2$ are left. Parity conservation gives $P(2\pi^0)=+ = P(\bar{p}p)=(-1)^{l+1}$. Hence, l=odd, leaving only $^3\mathrm{P}_2$.

3.20. The G-parity is positive, because it is conserved and is such in the final state. As $G=C(-1)^I$, if I=0 then C=+, i. e. $C=(-1)^l=+1$. Then l=even. If I=1, then C=–, i.e. $(-1)^l=-1$. We have l=odd.

3.21. It is convenient to prepare a table with the possible values of the initial J^{PC} and of the final

l^{CP} with l=J to satisfy angular momentum conservation. Only the cases with the same parity and charge conjugation are allowed. Recall that $P(\bar{p}p)=(-1)^{l+1}$ and $C(\bar{p}p)=(-1)^{l+s}$.

	$^1S_0$	$^3S_1$	$^1P_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	$^3D_1$	$^3D_2$	$^3D_3$
J^{PC}	0^{-+}	1^{--}	1^{+-}	0^{++}	1^{++}	2^{++}	2^{-+}	1^{--}	2^{--}	3^{--}
l^{PC}	0^{++}	1^{--}	1^{--}	0^{++}	1^{--}	2^{++}	2^{++}	1^{--}	2^{++}	3^{--}
		Y		Y		Y		Y		Y

In conclusion 1. $^1S_0$; 2. $^3S_1$, $^3D_1$ e 3. $^3P_2$.

3.22. Given the quark content, Λ_c has electric charge +1. Since the processes is strong, a) and c) are forbidden by charm conservation, while b) is allowed. d) is violates charge conservation.

3.23. Λ_b is neutral. a) violates charm and beauty, b) and c) are allowed, d) violates beauty, e) violates baryon number.

3.26.

1. The minimum velocity is $\beta_{\min}=\frac{1}{n}=0.75$.

2. The minimum kinetic energy for electrons is

$$E_{kin,\min}(e)=m_e\left(\frac{1}{\sqrt{1-\beta_{\min}^2}}-1\right)=0.511\times 0.51=0.26\ \text{MeV}$$

and for K^+: $E_{kin,\min}(K^+)=m_K\left(\frac{1}{\sqrt{1-\beta_{\min}^2}}-1\right)=497.6\times 0.51=254\ \text{MeV}$

3. In the decay $p\rightarrow e^+ +\pi^0$, the CM kinetic energy of the e^+ is

$$E_{kin}(e^+)=\frac{m_p^2+m_e^2-m_{\pi^0}^2}{2m_p}-m_e=\frac{938.3^2+0.51^2-0.135^2}{2\times 938.3}-0.51=469\ \text{MeV}$$, above threshold

In the decay $p\rightarrow K^+ +\nu$, the CM kinetic energy of the K is

$$E_{kin}(K^+)=\frac{m_p^2+m_K^2-m_\nu^2}{2m_p}-m_K=\frac{938.3^2+497.6^2}{2\times 938.3}-497.6=104\ \text{MeV}$$, below threshold

3.27. (a) forbidden by lepton number, (b) forbidden by angular momentum and lepton number, (c) forbidden by charge conjugation, (e) allowed, (f) forbidden by baryon and lepton numbers, (g) forbidden by angular momentum and lepton number.

3.28. (a) X must have charge Q=0 and strangeness S=+1; it is a K^0; (b) X must have charge Q=0 and lepton number $\mathcal{L}_e$ =–1, it is a $\bar{\nu}_e$; (c) X must have Q=0; the reaction being weak, strangeness does not need to be conserved; it is a π^0.

3.29. The third component of the initial isospin is $I_{z,initial}=1+\frac{1}{2}=\frac{3}{2}$, hence the total isospin must be $I=\frac{3}{2}$. For the K^+K^+ we have I=1, I_z=1. Hence for the Ξ^0 may have $I_\Xi=\frac{1}{2}$ or $I_\Xi=\frac{3}{2}$.

The third component is $I_{\Xi,z} = +\frac{1}{2}$.

3.30. The total energy and its square are: $\sqrt{s} = m_p + m_{\pi^-} = 0.9383 + 0.1396 = 1.08$ GeV and $s = 1.162$ GeV^2.

The energy of the photon is $E_\gamma = \frac{s - m_n^2}{2\sqrt{s}} = \frac{1.162 - 0.883}{2 \times 1.08} = 0.129$ GeV.

The kinetic energy of the neutron is $T_n = \sqrt{s} - E_\gamma - m_n = m_p - m_n + m_\pi - E_\gamma$, which is a very small quantity, expressed as a difference between large quantities. It is then convenient to consider the nonrelativistic expression of the kinetic energy:

$$T_n = \frac{p_n^2}{2m_n} = \frac{E_\gamma^2}{2m_n} = \frac{130^2}{2 \times 939.6} = 9 \text{ MeV}.$$

3.31. The beam energy is enough to produce strange particles but not for heavier flavours. In order to conserve strangeness the *V*^{0}s must be a K^0 and a Λ. The simplest reaction is $\pi^+ + p \to \pi^+ + \pi^+ + K^0 + \Lambda$.

We calculate the mass of each V^0 assuming in turn it to be the K^0 or the a Λ.

If 1 is a Λ,

$$M^2 = m_p^2 + m_\pi^2 + 2\sqrt{p_{1+}^2 + m_p^2}\sqrt{p_{1-}^2 + m_\pi^2} - 2p_{1+}p_{1-}\cos\theta_1 =$$

$$= 0.938^2 + 0.139^2 + 2 \times 1.02 \times 1.905 - 2 \times 0.4 \times 1.9 \times \cos 24.5° = 3.38 \text{ GeV}^2$$

or M=1.83 GeV not compatible with being a Λ.

If 1 is a K^0

$$M^2 = 2m_\pi^2 + 2\sqrt{p_{1+}^2 + m_\pi^2}\sqrt{p_{1-}^2 + m_\pi^2} - 2p_{1+}p_{1-}\cos\theta_1 =$$

$$= 0.04 + 2 \times 0.424 \times 1.905 - 2 \times 0.4 \times 1.9 \times \cos 24.5° = 0.246 \text{ GeV}^2$$

or M=0.495 GeV compatible, within the errors, with the mass of the K^0

If 2 is a Λ,

$$M^2 = m_p^2 + m_\pi^2 + 2\sqrt{p_{2+}^2 + m_p^2}\sqrt{p_{2-}^2 + m_\pi^2} - 2p_{2+}p_{2-}\cos\theta_2 =$$

$$= 0.938^2 + 0.139^2 + 2 \times 1.20 \times 0.29 - 2 \times 0.75 \times 0.25 \times \cos 22° = 1.59 - 0.35 = 1.24 \text{ GeV}^2$$

or M=1.11 GeV compatible, within the errors, with the mass of the Λ.

If 2 is a K^0

$$M^2 = 2m_\pi^2 + 2\sqrt{p_{2+}^2 + m_\pi^2}\sqrt{p_{2-}^2 + m_\pi^2} - 2p_{2+}p_{2-}\cos\theta_2 =$$

$$= 0.04 + 2 \times 0.76 \times 0.29 - 0.35 = 0.138 \text{ GeV}^2$$

or M=0.371 GeV incompatible, within the errors, with the mass of the K^0.

4.2. For the ω, $G = C(-1)^I = (-1)(-1)^0 = -1$. For the ϕ, $G = C(-1)^I = (-1)(-1)^0 = -1$. The K is not an eigenstate of G. For the η, $G = C(-1)^I = (+1)(-1)^0 = +1$.

4.3. The ρ decays strongly into 2π, hence G=+. The possible values of its isospin are 0, 1 and 2. In the three cases the Clebsch-Gordan coefficients are $\langle 1,0|1,0;1,0\rangle = 0$, $\langle 0,0|1,0;1,0\rangle \neq 0$ and $\langle 2,0|1,0;1,0\rangle \neq 0$. Hence I=1.

Since I=1, the isospin wave function is antisymmetric. The spatial wave function must consequently be antisymmetric, i. e. the orbital momentum of the two π must be l=odd. The ρ spin is equal to l. $C=(-1)^l=-1$. $P=(-1)^l=-1$.

4.4. $\tau_{K*} = 1.3\times 10^{-23}$ s, $\gamma = E/m \simeq p/m = 10.1$, $d = \gamma c\tau = 39$ fm .

4.5. a. From the size of the resonance width Γ we infer that it decays by strong interaction. As a consequence, S, I, I_z and Y are conserved. Therefore $S(\Sigma)= S(\Lambda)+ S(\pi)$=1+0=+ 1 and $Y(\Sigma)= Y(\Lambda)+ Y(\pi)$=0+0=0. $\mathbf{I}(\Sigma)= \mathbf{I}(\Lambda)\otimes\mathbf{I}(\pi)=\mathbf{0}\otimes\mathbf{1}=\mathbf{1}$ and $I_z(\Sigma)= I_z(\Lambda)+ I_z(\pi)$=0+0=0.

b. $\mathbf{J}(\Sigma)= \mathbf{J}(\Lambda)\otimes\mathbf{J}(\pi)\otimes\mathbf{L}=\mathbf{1/2}\otimes\mathbf{0}\otimes\mathbf{1} \Rightarrow J=1/2$ or $J=3/2$. $P(\Sigma)= P(\Lambda)\, P(\pi)\, (-1)^L=(+)(-)(-)=+$.

4.6. a. The decay is strong b. The initial strangeness in the reaction $K^- + p \to \pi^- + \Sigma^+(1385)$ is S=–1. The strangeness of the Σ (1385) is S=–1. Since the isospin is conserved in the strong decay, the isospin of the Σ (1385) is equal to the isospin of the $\pi^+\Lambda$ system, i. e. is 1.

4.7. 1. Two equal bosons cannot be in an antisymmetric state; 2. $C(2\pi^0)$=+1; 3. the Clebsch Gordan coefficient $\langle 1,0;1,0|1,0\rangle = 0$.

4.8. The charge conjugation of the final state $\pi^+\pi^-$ is $C=(-1)^l=(-1)^J$, i.e. ρ^0 has C=–; the f^0 has C=+. The system $\pi^0\gamma$ has C=(+)(–)=–. Hence $f^0\to\pi^0\gamma$ is forbidden.

4.9. $R=\Gamma\left(K^{*+} \to K^0 + \pi^+\right)/\Gamma\left(K^{*+} \to K^+ + \pi^0\right) = 1/2$ if I_{K*}=3/2. R =2 if I_{K*}=1/2.

4.10. $\Gamma\left(K^- p\right)/\Gamma\left(\bar{K}^0 n\right) = 1$. $\Gamma\left(\pi^-\pi^+\right)/\Gamma\left(\bar{K}^0 n\right) = 0$, because the decay into $\pi^+\pi^-$ would violate baryon number and strangeness.

4.11. It is useful to prepare a table with the quantum numbers of the relevant states

	$\bar{p}p\,^3S_1$	$\bar{p}p\,^3S_1$	$\bar{p}p\,^1S_0$	$\bar{p}p\,^1S_0$	$\bar{p}n\,^3S_1$	$\bar{p}n\,^1S_0$
J^P	1^-	1^-	0^-	0^-	1^-	0^-
C	–	–	+	+	X	X
I	0	1	0	1	1	1
G	–	+	+	–	+	–

$\bar{p}n \to \pi^-\pi^-\pi^+$. Since G=–1 in the final state, there is only one possible initial state, i.e. $^1S_0$

$$|\bar{p},n\rangle = |1,-1\rangle = \frac{1}{\sqrt{2}}|1,0;1,-1\rangle - \frac{1}{\sqrt{2}}|1,-1;1,0\rangle = \frac{1}{\sqrt{2}}\left|\rho^0;\pi^-\right\rangle - \frac{1}{\sqrt{2}}\left|\rho^-;\pi^0\right\rangle$$

hence $R\left(\bar{p}n \to \rho^0\pi^-\right)/R\left(\bar{p}n \to \rho^-\pi^0\right) = 1$.

$$|\bar{p},p\rangle = |1,0\rangle = \frac{1}{\sqrt{2}}\left|\rho^-;\pi^+\right\rangle + 0\frac{1}{\sqrt{2}}\left|\rho^0;\pi^0\right\rangle - \frac{1}{\sqrt{2}}\left|\rho^+;\pi^-\right\rangle$$

hence $R\left(\bar{p}p(I=1) \to \rho^+\pi^-\right) : R\left(\bar{p}p(I=1) \to \rho^0\pi^0\right) : R\left(\bar{p}p(I=1) \to \rho^-\pi^+\right) = 1:0:1$.

$$|\bar{p},p\rangle = |0,0\rangle = \frac{1}{\sqrt{3}}\left|\rho^-;\pi^+\right\rangle - \frac{1}{\sqrt{3}}\left|\rho^0;\pi^0\right\rangle + \frac{1}{\sqrt{3}}\left|\rho^+;\pi^-\right\rangle$$

hence $R\left(\bar{p}p(I=0) \to \rho^+\pi^-\right) : R\left(\bar{p}p(I=0) \to \rho^0\pi^0\right) : R\left(\bar{p}p(I=0) \to \rho^-\pi^+\right) = 1:1:1$.

4.12. The isospin wave function must be symmetrical, because the spatial wave function is such. Hence I = 0 or 2.

4.13. The matrix element $\mathcal{M}$ must be symmetric under the exchange of each pair of pions. Consequently:

1. if $J^P = 0^-$, $\mathcal{M}$ = constant. There are no zeros.
2. if $J^P = 1^-$, $\mathcal{M} \propto \mathbf{q}(E_1 - E_2)(E_2 - E_3)(E_3 - E_1)$; zeros on the diagonals and on the border.
3. if $J^P = 1^+$, $M \propto \mathbf{p}_1 E_1 + \mathbf{p}_2 E_2 + \mathbf{p}_3 E_3$; zero in the centre, where $E_1=E_2=E_3$; zero at $T_3=0$, where $\mathbf{p}_3=0$, $\mathbf{p}_2=-\mathbf{p}_1$; $E_2=E_1$.

4.14. Since the parity is positive, the orbital momentum l of the two nucleons must be even. The total angular momentum is J=1, the sum of the two spins can be s=0 or s=1. Hence we can have l=0 or l=2. The two possible states are $^3S_1$ and $^3D_1$.

4.15. A baryon can contain between 0 and 3 c valence quarks; therefore the charm of a baryon can be C=0, 1, 2, 3. Since the charge of c is equal to 2/3, the baryons with Q=+1 can have charm C=2 (*ccd*, *cc*s, *ccb*), C=1 (e.g. *cud*) or C=0 (e.g. *uud*). If Q=0, one c can be present, as in *cdd*, or none as in *udd*. Hence C=1 or C=0.

4.16. Since $\mathcal{B}$=1 the particle is a baryon. Therefore the valence quarks are three. Since the charge Q=+1, two quarks are up-type, one is down-type. Since C=1, one up-type quark is c. Since S=0, B=0, T=0, the other two quarks are u and d. The state is *udc*.

4.17. *sss*, *uuc*, *usc*, *ssc*, *udb*.

4.18. $c\bar{d}$, $\bar{c}u$, $u\bar{b}$, $c\bar{b}$.

4.22. $\tau_{J/\psi} = \frac{1}{\Gamma} == \frac{1}{0.091 \times 1.52 \times 10^{21}\text{s}^{-1}} = 7.2 \times 10^{-21}\text{s}$.

The distance travelled in a lifetime is $l_{\text{lab}} = \gamma\tau_{J/\psi}\beta c = \tau_{J/\psi}\frac{p}{M}c = 3.5 \times 10^{-12}\,\text{m}$.

Let E_e be the energy and $p_e \approx E_e$ the momentum of the electron. From $E_J = 2E_e$ and $p_J = 2p_e\cos\theta_e$, we have $E_e = \frac{E_J}{2} = \frac{1}{2}\sqrt{p_J^2 + m_J^2} = 2.94\ \text{GeV}$.

From $m_e^2 = \left(\frac{E_J}{2}\right)^2 - \left(\frac{p_J}{2\cos\theta_e}\right)^2$, we have $\cos\theta_e = \frac{p_J}{\sqrt{E_J^2 - 4m_e^2}} \approx \frac{p_J}{E_J} = 0.85$; i. e. $\theta_e = 31.8^\circ$.

For p_J=50 GeV, $\theta_e = 3.6^\circ$.

4.23. $\gamma = E/m = 20/1.86 = 10.7$. The condition $I = I_0 e^{-\frac{t}{\gamma\tau}} > 0.9 I_0$ gives $t < \gamma\tau \ln\left(\frac{1}{0.9}\right)$. We need to resolve distances $d = ct < 139\ \mu\text{m}$.

Possible instruments: bubble chambers, emulsions, Silicon microstrips.

4.24. We start from $\sigma(E) = \frac{3\pi}{E^2}\frac{\Gamma_e\Gamma_f}{(E - M_R)^2 + (\Gamma/2)^2} = \frac{12\pi\Gamma_e\Gamma_f}{\Gamma^2}\frac{1}{E^2}\frac{1}{[2(E - M_R)/\Gamma]^2 + 1}$.

In the neighbourhood of the resonance peak the factor $1/E^2$ varies only slowly, compared to the resonant factor, and we can approximate it with the constant $1/M_R^2$, i.e.

$$\int_{-\infty}^{+\infty}\sigma(E)dE = \frac{12\pi\Gamma_e\Gamma_f}{\Gamma^2}\int_{-\infty}^{+\infty}\frac{1}{E^2}\frac{1}{\left[2(E-M_R)/\Gamma\right]^2+1}dE \simeq \frac{12\pi\Gamma_e\Gamma_f}{\Gamma^2 M_R^2}\int_{-\infty}^{+\infty}\frac{1}{\left[2(E-M_R)/\Gamma\right]^2+1}dE .$$

Setting $\tan\theta = \frac{2(E-M_R)}{\Gamma}$, we have

$$\int_{-\infty}^{+\infty}\sigma(E)dE = \frac{12\pi\Gamma_e\Gamma_f}{\Gamma^2 M_R^2}\int_{-\infty}^{+\infty}\frac{1}{\tan^2\theta+1}dE = \frac{12\pi\Gamma_e\Gamma_f}{\Gamma^2 M_R^2}\int_{-\infty}^{+\infty}\cos^2\theta dE = \frac{12\pi\Gamma_e\Gamma_f}{\Gamma^2 M_R^2}\int_{-\pi/2}^{+\pi/2}\cos^2\theta\frac{dE}{d\theta}d\theta .$$

We find that $\frac{dE}{d\theta} = \frac{dE}{d\tan\theta}\frac{d\tan\theta}{d\theta} = \frac{\Gamma}{2}\frac{1}{\cos^2\theta}$, obtaining

$$\int_{-\infty}^{+\infty}\sigma(E)dE = \frac{6\pi\Gamma_e\Gamma_f}{\Gamma M_R^2}\int_{-\pi/2}^{+\pi/2}d\theta = \frac{6\pi^2\Gamma_e\Gamma_f}{\Gamma M_R^2} .$$

4.25. The total energy is $E = E^- + E^+ = 12.1$ GeV, the total momentum is $p = p^- - p^+ = 5.9$ GeV.

We have $E = 2E_B$; $p = 2p_B\cos\theta_B$ and $E_B = E/2 = 6.05$ GeV; $p_B = \sqrt{E_B^2 - m_B^2} = 2.96$ GeV.

The Lorentz factors are $\beta = \frac{p_B}{E_B} = \frac{2.96}{6.05} = 0.49$ $\gamma = \frac{E_B}{m_B} = \frac{6.05}{5.279} = 1.146$. The distance travelled in a lifetime is $l = \beta\gamma\tau_B c = 0.49\times1.146\times1.53\times10^{-12}\times3\times10^8 = 258\ \mu$m.

From $m_B^2 = E_B^2 - p_B^2 = \left(\frac{E}{2}\right)^2 - \left(\frac{p}{2\cos\theta_B}\right)^2$, we obtain $\cos\theta_B = \sqrt{\frac{p^2}{E^2-4m_B^2}} = 0.998$ and θ_B =3.6°.

4.28.

(a) forbidden by strangeness conservation, (b) allowed, (c) forbidden by charge conservation, (d) forbidden by energy conservation, (e) forbidden by strangeness conservation, (f) forbidden: strangeness, isospin and its third component are not conserved, (g) forbidden: strangeness, isospin and its third component are not conserved.

4.29. (a) forbidden by strangeness conservation, (b) forbidden by strangeness conservation, (c) allowed, (d) allowed, (e) forbidden by charge, strangeness, isospin and I_z, (f) forbidden by strangeness, isospin and I_z, (g) forbidden by strangeness.

4.30. *G*=+, because there are overall 4*π*.

The isospin cannot be 1, because the Clebsch-Gordan coefficient $\langle 1,0;1,0|1,0\rangle = 0$. It may then be *I* = 0 or *I* = 2.

C=+ because the two particles are identical. Check: $G=C(-1)^I=+$

The two particles are identical bosons, hence *L* must be even, *L*=0, 2, 4,..

The spin wave function must be symmetrical too, hence *S*=0, 2

It can be

J=0, with L=0, S=0 and with L=2, S=2

J=1 with L=2, S=2

J=2 with L=0, S=2, with L=2, S=0, with L=2, S=2 and with L=4, S=2.

4.31. G=–, because there are overall 3π.

The isospin cannot be 1, because the Clebsch-Gordan coefficient $\langle 1,0;1,0|1,0\rangle = 0$. It may then be $I = 0$ or $I = 2$.

Both ρ^0 and π^0 are eigenstates of C with C=– and C=+ respectively. Hence C=–, independently on L. Check: $G=C(-1)^I$=–

The total spin can be S=0 or S=1

There are no symmetry restrictions on L because the two particles are different.

J=0 with L=S= 0 or 2

J=1 with L=0, S= 1, or L=1, S= 0, or L=1, S= 1, or L=2, S=1

4.32. In order to conserve beauty the $\Lambda_b(udb)$ must be produced with a meson containing $\bar{b}$, a B. The reaction is $\pi^- p \to \Lambda_b B^0$.

The threshold pion energy is $E_\pi = \frac{\left(m_{\Lambda_b} + m_{B^0}\right)^2}{2m_p} = \frac{(5.624 + 5.297)^2}{2 \times 0.938} = 63.6\text{ GeV}$.

4.33. In order to conserve charm the $\Sigma_c^{++}(uuc)$ must be produced with a meson containing $\bar{c}$, a D^-. The reaction is $\pi^- p \to \Sigma_c^{++} D^- \pi^-$.

The threshold pion energy is $E_\pi = \frac{\left(m_{\Sigma_c^{++}} + m_{D^0} + m_\pi\right)^2}{2m_p} = \frac{(2.452 + 1.869 + 0.139)^2}{2 \times 0.938} = 10.6\text{ GeV}$

4.34. With a π^- beam, to conserve strangeness and charm we need to produce together with $\Omega_c^0(ssc)$ one particle containing $\bar{c}$, say $D^-(d\bar{c})$ and two containing $\bar{s}$, say, to conserve also the charge, $K^+(u\bar{s})$ and $K^0(d\bar{s})$. The reaction is $\pi^- p \to \Omega_c^0 D^- K^+ K^0$. Its threshold is

$$E_\pi = \frac{\left(m_{\Omega_c^0} + m_{D^-} + m_{K^+} + m_{K^0}\right)^2}{2m_p} = \frac{(2.698 + 1.869 + 0.494 + 0.498)^2}{2 \times 0.938} = 16.5\text{ GeV}$$

With a K^- beam the initial strangeness is S=+1, hence only one K meson needs to be produced. The reaction is $K^- p \to \Omega_c^0 D^- K^+$. Its threshold is

$$E_K = \frac{\left(m_{\Omega_c^0} + m_{D^-} + m_{K^+}\right)^2}{2m_p} = \frac{(2.698 + 1.869 + 0.494)^2}{2 \times 0.938} = 13.7\text{ GeV}$$

With a K^{-+} beam the initial strangeness is S=–1, hence three K mesons must be produced. The reaction is $K^+ p \to \Omega_c^0 D^- K^+ K^+ K^+$. Its threshold is

$$E_K = \frac{\left(m_{\Omega_c^0} + m_{D^-} + 3m_{K^+}\right)^2}{2m_p} = \frac{(2.698 + 1.869 + 3 \times 0.494)^2}{2 \times 0.938} = 19.5 \text{ GeV}.$$

5.1. Kinetic and potential energies are of the same order for the virial theorem. We then evaluate the kinetic energy of the electron in the hydrogen atom to be $E_{ke} \approx 10$ eV$<<m_e \approx 5\times10^5$eV. Therefore the motion is non-relativistic, and we have

$$\beta = \sqrt{\frac{2E_{ke}}{m_e}} = \sqrt{\frac{2\times 10}{5\times 10^5}} = 6\times 10^{-3}.$$

For a proton in a nucleus we estimate $E_{kp} \approx 1$ MeV $<< m_p \approx 938$ MeV. Again the motion is non-relativistic, and we have

$$\beta = \sqrt{\frac{2E_{kp}}{m_p}} = 4.4\times 10^{-2}.$$

For a quark in a nucleon we estimate $E_{kq} \approx \lambda \approx 400$ MeV. Since $m_q \approx 4$ MeV, the motion is relativistic with $\gamma = E_{kq}/m_q \approx 100$ and we have

$$\beta \simeq 1 - \frac{1}{2}\gamma^{-2} = 0.99995.$$

5.2. Since the speeds are small enough, we can use non-relativistic concepts and expressions. The electron potential energy, which is negative, becomes smaller with its distance r from the proton as $-1/r$. The closer the electron is to the proton, the better is its position defined and consequently the larger is the uncertainty of its momentum p. Actually, the larger the uncertainty of p the larger is its average value and, with it, the electron kinetic energy. The radius of the atom is the distance at which the sum of potential and kinetic energies is minimum.

Due to its large mass, we consider the proton to be immobile. At the distance r the energy of the electron is

$$E = \frac{p^2}{2m_e} - \frac{1}{4\pi\varepsilon_0}\frac{q_e^2}{r}.$$

The uncertainty principle dictates $pr = \hbar$ and we have $E = \frac{\hbar^2}{2m_e r^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_e^2}{r}$.

To find the minimal radius a we set $\left(\frac{dE}{dr}\right)_a = 0 = -\frac{\hbar^2}{m_e a^3} + \frac{1}{4\pi\varepsilon_0}\frac{q_e^2}{a^2}$, obtaining

$a = \frac{4\pi\varepsilon_0\hbar^2}{m_e q_e^2} = 52.8$ pm, that is the Bohr radius.

5.3. The unperturbed energies for the levels n=2 and n=3 are: $E_2 = -3.40$ eV and $E_3 = -1.51$ eV. In the (5.8) the factor depending on j is $1/(j+1/2)$. The difference between its values for j=3/2 and j=1/2 is $\frac{1}{3/2+1/2} - \frac{1}{1/2+1/2} = -\frac{1}{2}$, and e have $E_{n,3/2} - E_{n,1/2} = E_n \frac{\alpha^2}{n}\frac{1}{2}$.

Finally we obtain $E_{2,3/2} - E_{2,1/2} = 45.2\ \mu$eV e $E_{3,3/2} - E_{3,1/2} = 13.4\ \mu$eV.

5.4. The distance between the vertices is $R \approx 1/\Delta E$, where ΔE is the energy violation at, say, the first vertex as computed assuming momentum conservation. In the rest frame of the electron it is $R_0 \approx 1/m_e$, in the CM reference it is $R \approx 1/2E_e \approx 1/E_e$. Their ratio is: $\frac{R_0}{R} = \frac{E_e}{m_e} = \gamma$, which is the Lorentz factor between the two frames, as it must be.

5.6. At the next to the tree-level order in the t-channel there are the eight diagrams in the figure

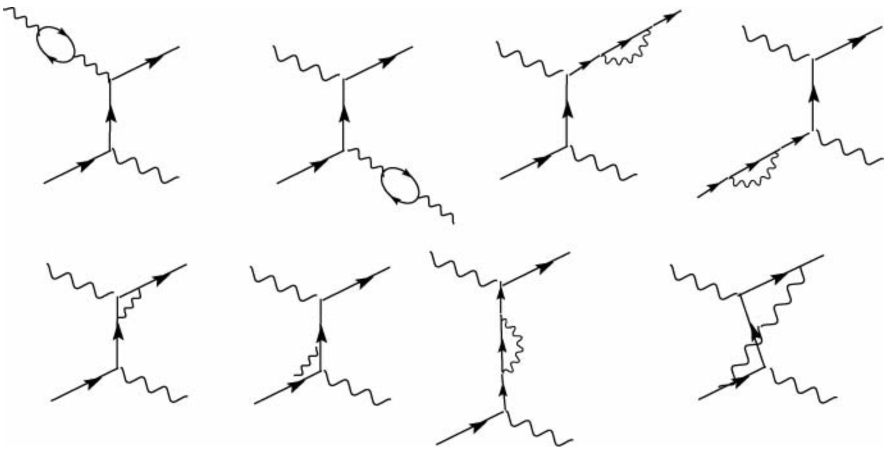

Fig. S.3

There are as many diagrams in the s-channel. The last one is

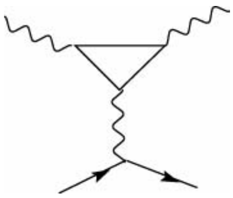

Fig. S.4

5.7. 145 nb, 9.1 nb, 0.97 nb e 0.01 nb, $\int_{\pi/2}^{\pi}\left((1+\cos^2\theta\right)\sin\theta d\theta \Big/ \int_{0}^{\pi}\left((1+\cos^2\theta\right)\sin\theta d\theta = 1/2$.

5.8. $\sigma_{m_\psi}\left(e^+e^- \to \mu^+\mu^-\right) = \frac{12\pi}{m_\psi^2}\frac{\Gamma_e^2}{\Gamma^2} = \frac{12\pi}{\left(3.097\right)^2}\left(5.9\times10^{-2}\right)^2 \times 389 \ \left(\mu\text{b/GeV}^{-2}\right) = 5.3\ \mu\text{b}$.

$\sigma_{m_\psi}\left(e^+e^- \to \text{hadrons}\right) = \sigma_{m_\psi}\left(e^+e^- \to \mu^+\mu^-\right)\frac{87.7\%}{5.9\%} = 84\ \mu\text{b}$.

5.9. From (4.67) we have $\int\sigma_{\mu\mu}\left(E\right)dE = \frac{6\pi^2\Gamma_\mu^2}{m_\Upsilon^2\Gamma} = 8$ nb MeV$=2.08\times10^{-11}$ MeV^{-1}.

$\Gamma_\mu^2 = \frac{m_\Upsilon^2\left(\text{MeV}^2\right)\Gamma\left(\text{MeV}\right)}{6\pi^2}\times 2.08\times10^{-11}\ \text{MeV}^{-1} = 1.7\times10^{-6}\ \text{MeV}^2$, that gives

$\Gamma_\mu = 1.3$ keV and $\Gamma_h = \Gamma_\mu \times \frac{310}{8} = 50$ keV .

5.12. The two contributions are proportional to $z_q = 3\left(\frac{4}{9}2 + \frac{1}{9}2\right) = 3.3$ and to $z_l = 3$. Their ratio is $z_q / z_l = 1.1$.

5.13.

1. The reaction is $\gamma + {}^{16}\mathrm{O} \to e^+ + e^- + {}^{16}\mathrm{O}$

The CM energy squared is $s = \left(E_\gamma + m_O\right)^2 - p_\gamma^2 = m_O^2 + 2m_O E_\gamma$. The threshold condition is $s = \left(2m_e + m_O\right)^2 \simeq m_O^2 + 4m_e m_O$. Hence $E_\gamma = 2m_e = 1.02$ MeV

2. The reaction is $\gamma + e^- \to e^+ + 2e^-$.

The CM energy squared is $s = \left(E_\gamma + m_e\right)^2 - p_\gamma^2 = m_e^2 + 2m_e E_\gamma$. The threshold condition is $s = 9m_e^2$. Hence $E_\gamma = 4m_e = 2.04$ MeV.

3. The reaction is $\gamma + p \to \mu^+ + \mu^- + p$.

The CM energy squared is $s = \left(E_\gamma + m_p\right)^2 - p_\gamma^2 = m_p^2 + 2m_p E_\gamma$. The threshold condition is $s = \left(2m_\mu + m_p\right)^2 \simeq m_p^2 + m_\mu^2 + 2m_\mu m_p$. Hence $E_\gamma = 2m_\mu + \dfrac{m_\mu^2}{2m_p} = 2\times 106 + \dfrac{106^2}{2\times 938} = 224$ MeV.

4. $m_{ee}^2 = \left(E_+ + E_-\right)^2 - \left(\mathbf{p}_+ - \mathbf{p}_-\right)^2 = 2m_e^2 + 2\left(E_+E_- - p_+p_-\cos\theta\right) \simeq 2m_e^2 + 2E_+E_-\left(1-\cos\theta\right)$. The minimum mass of the pair is for opening angle θ=0 and its value is $\sqrt{2}m_e$.

5.14. In the case of the electron the Bohr radius is $a_e = \dfrac{4\pi\varepsilon_0\hbar^2}{q_e^2 m_e} = \dfrac{\hbar}{\alpha m_e} = 53\ \mathrm{pm}$, which is inversely proportional to the electron mass. To be precise, the reduced mass must be considered. The reduced mass of a system composed by a proton and a particle of mass m is $m_R \equiv \dfrac{mm_p}{m+m_p}$. We have: $m_{R\mu} = 95$ MeV, $m_{R\pi} = 121$ MeV, $m_{RK} = 325$ MeV, $m_{R\bar{p}} = 469$ MeV and $a_\mu = a_e \dfrac{m_e}{m_{R\mu}} = 280$ fm, $a_\pi = 220$ fm, $a_K = 82$ fm, $a_{\bar{p}} = 56$ fm.

b) It is 13.6 eV for an electron and is proportional to the (reduced) mass. Hence $13.6\dfrac{121}{0.5} = 3.3$ keV.

5.15. The two pions in the final state have l=1; the spatial wave function is odd. The total wave function must be even, hence their isospin must be $I_{\pi\pi}$=1. The decay violates the isospin by $\Delta I = 1$. Moreover I_z is conserved. The decay cannot be strong, but can be electromagnetic.

5.17.

(a) $C(\eta) = C(\pi\pi)C(\gamma)$ or $+ = -C(\pi\pi)$, hence $C(\pi\pi) = -$ and $l_{\pi\pi}$=odd, with minimum value =1. The total wave function must be even. Being the spatial part odd, the isospin part must be odd, hence $I_{\pi\pi}$=1. The isospin violation is ΔI =1.

(b) $C(\omega) = C(\pi\pi)C(\gamma)$ or $- = -C(\pi\pi)$, hence $C(\pi\pi) = +$ and $l_{\pi\pi}$=even, with minimum value =0. Being the spatial part even, the isospin part must be even, hence $I_{\pi\pi}$=0 or 2. Electromagnetic interaction has ΔI =0 or 1. Hence $I_{\pi\pi}$=0 only.

(c) As (b) but with $I_{\pi\pi}$=0 and 2 both allowed.

A $\pi^0\pi^0$ cannot be in $|I, I_z\rangle = |1,0\rangle$: $\eta \to \pi^0\pi^0\gamma$ forbidden, $\omega \to \pi^0\pi^0\gamma$ allowed, $\rho^0 \to \pi^0\pi^0\gamma$ allowed.

5.18. From the data of the problem we obtain $R \equiv \dfrac{J/\psi \to \rho^0\pi^0}{J/\psi \to \rho\pi} = 0.33 \pm 0.05$.

With the help of the tables of the Clebsch-Gordan coefficients we have R=1/3 for I=0, R=0 for I=1 and R=2/3 for I=2. Hence I=0.

5.19.

(a) $s = \left(E_\gamma + E_e\right)^2 - \left(\mathbf{p}_\gamma + \mathbf{p}_e\right)^2 = m_e^2 + 2E_\gamma E_e - 2\mathbf{p}_\gamma \bullet \mathbf{p}_e = m_e^2 + 2E_\gamma E_e$,

because, in average $\mathbf{p}_\gamma \cdot \mathbf{p}_e = 0$

And $s = \left(E_{\gamma FT} + m_e\right)^2 - \mathbf{p}_{\gamma FT}^2 = m_e^2 + 2E_{\gamma FT} m_e$

Hence $E_{\gamma FT} = E_\gamma \dfrac{E_e}{m_e} = 0.25 \times 10^{-12} \dfrac{10^2}{0.5 \times 10^{-3}} = 5 \times 10^{-8}$ GeV=50 eV

(b) $\lambda = \dfrac{1}{\sigma\rho} = \dfrac{1}{7.9 \times 10^{-30} \times 3 \times 10^8} \simeq 4.2 \times 10^{20}$ m

$f = \dfrac{N_e c}{\lambda} \simeq \dfrac{1.6 \times 10^{12} \times 3 \times 10^8}{4.2 \times 10^{20}}$ =1.2 s^{-1}

(c) The kinematics is the same as that of problem 2.4

$$E_{\gamma f} = \frac{s - m_e^2}{2\left(E_T - p_T\right)}$$

$$E_T - p_T \simeq \frac{m_e^2}{2E_{ei}} + 2E_{\gamma i} \simeq \frac{m_e^2}{2E_{ei}} = \frac{0.5^2 \times 10^{-6}}{2 \times 10^2} = 1.25 \times 10^{-9} \text{ GeV}$$

$$s - m_e^2 = 4E_{\gamma i}E_{ei} = 4 \times 0.25 \times 10^{-12} \times 10^2 \text{ GeV}^2 = 10^{-10} \text{ GeV}^2$$

$$E_{\gamma f} = \frac{s - m_e^2}{2\left(E_T - p_T\right)} = \frac{10^{-10}}{2.5 \times 10^{-9}} = 4 \times 10^{-2} \text{ GeV=40 MeV}$$

6.2. The first case is below charm threshold, hence $R(u, d, s)$=2; the second case is above the charm threshold and below the beauty one, hence $R(u, d, s, c)$=10/3=3.3

6.3. Assuming a typical momentum transversal to the jet of 1 GeV, the opening angle is $2\dfrac{1}{\sqrt{s}/2} = 0.2 = 11.5°$.

The ratio of the counting rates is, from (6.8), $\dfrac{1+\cos^2 90°}{1+\cos^2 30°} = 0.57$.

6.4. $\nu = \dfrac{Q^2}{2m_p x} = 62.5$ GeV, $E' = E - \nu = 37.5$ GeV.

6.5. From (6.12) with W=m_p, $2m_p\nu = Q^2$ follows and then from (6.16) we have x=1. Using (6.11) we then obtain $2m_p\nu = Q^2 = 2EE'(1-\cos\theta)$, and then (1.60) because $\nu = E - E'$.

6.6. $\nu_\mu + d \to \mu^- + u$; $\quad \nu_\mu + s \to \mu^- + u$; $\quad \bar{\nu}_\mu + u \to \mu^+ + d$;

$\bar{\nu}_\mu + \bar{d} \to \mu^+ + \bar{u}$; $\bar{\nu}_\mu + \bar{s} \to \mu^+ + \bar{u}$; $\nu_\mu + \bar{u} \to \mu^- + \bar{d}$.

6.8. For every *x*, the momentum transfer Q^2 varies from a minimum to a maximum value when the electron scattering angle varies from 0° to 180°. From Eq. (6.11) and (6.14) that are valid in the L frame and (6.16) we obtain $Q^2 = \dfrac{2E^2\left(1-\cos\theta_f\right)}{1+\dfrac{E}{xm_p}\left(1-\cos\theta_f\right)}$.

Clearly, we have Q^2=0 in the forward direction (θ=0). The maximum momentum transfer is for background scattering (θ=180°), i. e. $Q^2_{\max} = \dfrac{4E^2}{1+\dfrac{2E}{xm_p}} \simeq 2Exm_p$.

For *E*=100 GeV, *x*=0.2 we have $Q^2_{\max} = 37.5\ \text{GeV}^2$, corresponding to a resolving power of 32 am.

6.9. $\sqrt{s} \simeq 2\sqrt{E_pE_e} = 313.7\ \text{GeV}$; $E_{e,f} = \dfrac{s}{2m_p} = 52.4\ \text{TeV}$. This is also the energy of the electron in the rest frame of the proton. We shall now work in this frame and use the results of the problem 6.8.

The maximum momentum transfer is for a given *x* is $Q^2_{\max} \simeq 2E_{e,f}xm_p$.

We have for *x*=0.4 $Q^2_{\max} = 4.2\times10^4\ \text{GeV}^2$, for *x*=0.1 $Q^2_{\max} = 10^4\ \text{GeV}^2$ and for *x*=0.01 $Q^2_{\max} = 10^3\ \text{GeV}^2$. The resolving power at *x*=0.4 is 1 am, about one thousands of the proton radius.

6.10. We write (5.37) with $\mu^2 = m_Z^2$ and $\alpha^{-1}\left(m_Z^2\right) = 129$, as $\alpha^{-1}\left(Q^2\right) = 129 - 0.71\times\ln\left(\dfrac{Q^2}{m_Z^2}\right)$.

Hence, $\alpha^{-1}\left(10^2\right) = 132$, $\alpha^{-1}\left(100^2\right) = 129$.

Eq. (6.69) with n_f=5 gives: $\alpha_s^{-1}\left(Q^2\right) = \dfrac{33-10}{12\pi}\ln\left(\dfrac{|Q|^2}{\lambda^2_{QCD}}\right) = 0.61\times\ln\left(\dfrac{|Q|^2}{0.04}\right)$.

Hence $\alpha_s^{-1}\left(10^2\right) = 4.8$ and $\alpha_s^{-1}\left(100^2\right) = 7.6$.

The ratios are $\alpha_s\left(10^2\right)/\alpha\left(10^2\right) = 27.5$ and $\alpha_s\left(100^2\right)/\alpha\left(100^2\right) = 16.9$.

6.11. $\Lambda_c^+ = udc$. (a) violates charm, (b) $D^- = d\bar{c}$ OK, (c) $\bar{D}^0 = u\bar{c}$, charm conserved but electric charge violated, (d) $D_s^- = s\bar{c}$, charm conserved but strangeness violated

6.12. Since *S*=–2 two quarks must be *s*, Since *Q*=0 the third must be *u*. The particle is *ssu*. Its isospin is *I*=1/2, with third component I_z=+1/2. It may be Ξ° (spin 1/2) or $\Xi^{*\circ}$ (spin 3/2).

6.13. The colour wave function is $\dfrac{1}{\sqrt{6}}\left[RGB - RBG + GBR - GRB + BRG - BGR\right]$, which is completely antisymmetric. Since the space wave function is symmetric, the product of the spin and isospin wave functions must be completely symmetric for any two-quark exchange. The system, *uud*, is obviously symmetric in the exchange within the *u* pair. Consider the *ud* exchange. The totally symmetric combination *uud*+*udu*+*duu* has isospin 3/2 and is not the proton.

The isospin 1/2 wave function contains terms that are antisymmetric under the exchange of the second and third quark, like *uud*–*udu*. We obtain symmetry by multiplying by a term with the same antisimmetry in spin, namely (↑↑↓–↑↓↑). We thus obtain a term symmetric under the exchange of the second and third quarks:

$$(u\uparrow)(u\uparrow)(d\downarrow)-(u\uparrow)(d\uparrow)(u\downarrow)-(u\uparrow)(u\downarrow)(d\uparrow)+(u\uparrow)(d\downarrow)(u\uparrow).$$

Similarly for the first two quarks we have

$$(u\uparrow)(u\uparrow)(d\downarrow)-(d\uparrow)(u\uparrow)(u\downarrow)-(u\downarrow)(u\uparrow)(d\uparrow)+(d\downarrow)(u\uparrow)(u\uparrow).$$

and for the first and third

$$(d\downarrow)(u\uparrow)(u\uparrow)-(u\downarrow)(d\uparrow)(u\uparrow)-(d\uparrow)(u\downarrow)(u\uparrow)+(u\uparrow)(d\downarrow)(u\uparrow)$$

In total we have 12 terms. We take their sum and normalise, obtaining

$$\frac{1}{\sqrt{12}}[2(u\uparrow)(u\uparrow)(d\downarrow)+2(d\downarrow)(u\uparrow)(u\uparrow)+2(u\uparrow)(d\downarrow)(u\uparrow)-(u\uparrow)(d\uparrow)(u\downarrow)-$$
$$-(u\uparrow)(u\downarrow)(d\uparrow)-(d\uparrow)(u\uparrow)(u\downarrow)-(u\downarrow)(u\uparrow)(d\uparrow)-(u\downarrow)(d\uparrow)(u\uparrow)-(d\uparrow)(u\downarrow)(u\uparrow)]$$

that is, as required, completely antisymmetric for the exchange of any pair.

6.18. $\Lambda_b^0 = udb$. (a) violates beauty and strangeness, (b) violates beauty and charm, (c) $B^0 = d\bar{b}$, OK; (d) $\Sigma_b^- = ddb$, $B^+ = u\bar{b}$ OK; (e) $\Sigma_b^+ = uub$, $B^- = \bar{u}b$, violates beauty.

6.19. The energy of the scattered electron is $E' = \dfrac{E}{1+\dfrac{E}{M}(1-\cos\theta)}$, where *M* is the target mass.

1. If the target is a proton $E' = \dfrac{1}{1+\dfrac{1}{0.938}(1-\cos 40^\circ)} = \dfrac{1}{1+\dfrac{1}{0.938}0.23} = 0.80$ GeV

2. If the target is a He nucleus $E' = \dfrac{1}{1+\dfrac{1}{4}0.23} = 0.94$ GeV

3. If the target is iron $E' = \dfrac{1}{1+\dfrac{1}{56}0.23} = 0.996$ GeV

6.20

a) $\sqrt{s} \simeq 2\sqrt{E_p E_e} = 300$ GeV

b) $Q^2 = 4E_e E_e^{'}\sin^2\dfrac{\theta}{2} = 4\times 28\times 223\times\sin^2 60^\circ = 18732\ \text{GeV}^2$

The four-momentum of the initial proton is $P_\mu = E_p, \mathbf{P}_p$ and the four-momentum transfer $q^\mu = (E_e^{'}-E_e),(\mathbf{p}_e^{'}\text{-}\mathbf{p}_e)$ and their (invariant) product

$$P_\mu q^\mu = E_p(E_e^{'}-E_e)-(\mathbf{p}_e^{'}-\mathbf{p}_e)\mathbf{P}_p = E_p(E_e^{'}-E_e)-p_e^{'}P_p\cos(180^\circ-\theta)+p_eP_p\cos(180^\circ)\simeq$$
$$\simeq E_e^{'}(E_p-2E_e)+E_e^{'}E_p\cos\theta = 223\times(820-2\times 28)+223\times 820\times\cos 120^\circ = 78942\ \text{GeV}^2$$

and $x = \frac{Q^2}{2P_\mu q^\mu} = \frac{18732}{2 \times 78942} = 0.11$

$$\nu = \frac{P_\mu q^\mu}{m_p} = \frac{78942}{0.938} = 84160 \text{ GeV}^2$$

$$W = m_p^2 + 2m_p\nu - Q^2 = 0.938^2 + 2 \times 0.938 \times 84160 - 18732 = 6.5 \times 10^4 \text{ GeV}^2 = \left(250 \text{ GeV}\right)^2$$

6.21. The energy of the beam is $E = \frac{E'}{1 - \frac{E'}{m_p}\left(1 - \cos\theta\right)}$. We have

$$E = \frac{12}{1 - \frac{12}{0.938}\left(1 - \cos 20^\circ\right)} = 52.5 \text{ GeV}$$

6.22.

a) The maximum transfer is for background scattering, namely θ=0. In this case (6.26) with (6.11) gives $Q^2_{\max} = 4EE' = \frac{4E^2 m_p}{m_p + 2E}$.

With E=15, GeV, $Q^2_{\max} = 27.3 \text{ GeV}^2$

From (6.31) we have the energy transfer to the proton $E - E' = \frac{Q^2_{\max}}{2m_p}$, which is the proton recoil kinetic energy $E_{k,rec} = \frac{Q^2_{\max}}{2m_p} = \frac{27.3}{2 \times 0.938} = 14.5 \text{ GeV}$

b) We have now $Q^2_{\max} = \frac{4E^2 M_{Fe}}{M_{Fe} + 2E} \simeq \left(2E\right)^2 = 0.04 \text{ GeV}^2$, as one would obtain just neglecting the recoil energy. This is given by $E_{k,rec} = \frac{Q^2_{\max}}{2M_{Fe}} = \frac{0.04}{2 \times 0.938} = 21 \text{ MeV}$, which is really a small fraction of the total recoil energy.

6.23.

a) The exchanged gluon is $g_2 = R\bar{B}$; the colour charges are $\left(\frac{\sqrt{\alpha_s}}{\sqrt{2}}\right)\left(\frac{\sqrt{\alpha_s}}{\sqrt{2}}\right) = \frac{\alpha_s}{2}$

b) The exchanged gluon is $g_2 = R\bar{B}$; the colour charges are $\left(\frac{\sqrt{\alpha_s}}{\sqrt{2}}\right)\left(-\frac{\sqrt{\alpha_s}}{\sqrt{2}}\right) = -\frac{\alpha_s}{2}$

c) There are two possible gluons to be exchanged: $g_7 = \frac{1}{\sqrt{2}}\left(R\bar{R} - G\bar{G}\right)$ and $g_8 = \frac{1}{\sqrt{6}}\left(R\bar{R} + G\bar{G} - 2B\bar{B}\right)$; the colour charges are $\left(\frac{\sqrt{\alpha_s}}{\sqrt{2}}\frac{1}{\sqrt{2}}\right)\left(-\frac{\sqrt{\alpha_s}}{\sqrt{2}}\frac{1}{\sqrt{2}}\right) = -\frac{\alpha_s}{2}\frac{1}{2}$ and $\left(\frac{\sqrt{\alpha_s}}{\sqrt{2}}\frac{1}{\sqrt{6}}\right)\left(-\frac{\sqrt{\alpha_s}}{\sqrt{2}}\frac{1}{\sqrt{6}}\right) = -\frac{\alpha_s}{2}\frac{1}{6}$. In total $-\frac{\alpha_s}{2}\frac{2}{3}$.

d) Force between two quarks is repulsive, between a quark and an antiquark is attractive

6.24.

a) The exchanged gluon is $g_6 = B\bar{G}$; the colour charges are $\left(\frac{\sqrt{\alpha_s}}{\sqrt{2}}\right)\left(\frac{\sqrt{\alpha_s}}{\sqrt{2}}\right) = \frac{\alpha_s}{2}$.

b) There are two possible gluons to be exchanged: $g_7 = \frac{1}{\sqrt{2}}\left(R\bar{R} - G\bar{G}\right)$ and $g_8 = \frac{1}{\sqrt{6}}\left(R\bar{R} + G\bar{G} - 2B\bar{B}\right)$; the colour charges are $\left(-\frac{\sqrt{\alpha_s}}{\sqrt{2}}\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{\alpha_s}}{\sqrt{2}}\frac{1}{\sqrt{2}}\right) = -\frac{\alpha_s}{2}\frac{1}{2}$ and $\left(\frac{\sqrt{\alpha_s}}{\sqrt{2}}\frac{1}{\sqrt{6}}\right)\left(-\frac{\sqrt{\alpha_s}}{\sqrt{2}}\frac{1}{\sqrt{6}}\right) = -\frac{\alpha_s}{2}\frac{1}{6}$. In total $-\frac{\alpha_s}{2}\frac{2}{3}$

c) There are two possible gluons to be exchanged: $g_7 = \frac{1}{\sqrt{2}}\left(R\bar{R} - G\bar{G}\right)$ and $g_8 = \frac{1}{\sqrt{6}}\left(R\bar{R} + G\bar{G} - 2B\bar{B}\right)$; the colour charges are $\left(-\frac{\sqrt{\alpha_s}}{\sqrt{2}}\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{\alpha_s}}{\sqrt{2}}\frac{1}{\sqrt{2}}\right) = -\frac{\alpha_s}{2}\frac{1}{2}$ and $\left(\frac{\sqrt{\alpha_s}}{\sqrt{2}}\frac{1}{\sqrt{6}}\right)\left(-\frac{\sqrt{\alpha_s}}{\sqrt{2}}\frac{1}{\sqrt{6}}\right) = -\frac{\alpha_s}{2}\frac{1}{6}$. In total $-\frac{\alpha_s}{2}\frac{2}{3}$.

d) Force between two quarks is repulsive, between a quark and an antiquark is attractive

6.26. Being the colour wave function symmetric, the product of the spin and space wave-functions must be symmetric.

The total spin and the corresponding symmetry are: *S*=0 symmetric, *S*=1 antisymmetric, *S*=2 symmetric.

The total orbital momentum can be *L*=0 symmetric, *L*=1 antisymmetric

Hence the following combinations are possible $S, L = 0{,}0$ or $1{,}1$ or $2{,}2$

Recall that $P = (-1)^L$, $C = (-1)^{L+S}$.

For *S*=0, $L = 0$, we have $J^{PC} = 0^{++}$

For *S*=1, $L = 1$, we have $J^{PC} = 0^{--}$, $J^{PC} = 1^{-+}$ and $J^{PC} = 2^{--}$

For *S*=2, $L = 0$, we have $J^{PC} = 2^{++}$

For *S*=2, $L = 2$, we have $J^{PC} = 0^{++}$, $J^{PC} = 1^{++}$, $J^{PC} = 2^{++}$, $J^{PC} = 3^{++}$, $J^{PC} = 4^{++}$.

6.27. The energy of the χ_c is: $E_{\chi_c} = m_\psi - E_\gamma$ and its momentum $p_\chi = p_\gamma$. Hence its mass is

$$m_\chi = \sqrt{\left(m_\psi - E_\gamma\right)^2 - p_\gamma^2} = \sqrt{m_\psi^2 - 2m_\psi E_\gamma} = \sqrt{3.686^2 - 2\times 0.26\times 3.686} = 3.42 \text{ GeV}$$

The photon emission brings out $J^P=1^-$. Hence the parity of the of the χ_c is + and possible values of its spin are *J* = 0, 1, 2. However, χ_c is observed to decay into $\pi^+\pi^-$, a system that can have $J^{PC}=0^{++}$, 1^{--}, 2^{++}. We are left with the two possibilities: $J^{PC}=0^{++}$, 2^{++}. The *G*-parity of the $\pi^+\pi^-$ system is +. From $G = C(-1)^I$ we find that the isospin of the χ_c may be *I*=0 or *I*=2. However, the decay of the ψ is an electromagnetic process, hence $\Delta I = 0{,}1$. In conclusion $I_{\chi_c} = 0$.

6.28. The states are: ${}^1P_1$: $J^{PC} = 1^{+-}$, ${}^3P_0$: $J^{PC} = 0^{++}$, ${}^3P_1$: $J^{PC} = 1^{++}$, ${}^3P_2$: $J^{PC} = 2^{++}$

Form the solution of problem 6.10 the three triplet states can decay into two gluons. The singlet decays into three gluons.

7.1. $K^{*+} \to K^0 + \pi^+$. We start by writing the valence quark compositions of all the particles, i. e. $(u\bar{s}) \to (d\bar{s}) + (u\bar{d})$ and then draw the diagram [figure a)]. Since it is a strong process we do not draw any gauge boson.

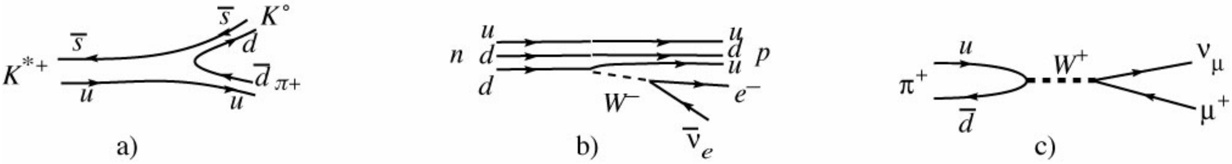

Fig. S.5

$n \to p + e^- + \bar{\nu}_e$. It is a weak process. In order to draw the diagram we consider two steps: the emission of a W, $(udd) \to (udu) + W^-$ and its decay $W^- \to e^- + \bar{\nu}_e$. [figure b)]

$\pi^+ \to \mu^+ + \nu_\mu$. We have $u\bar{d} \to W^+$ followed by $W^+ \to \mu^+ + \nu_\mu$. [figure c)]

7.2. $\pi^+(u\bar{d}) \to \pi^0(u\bar{u}) + e^+ + \nu_e$ [figure a)]. $\rho^+(u\bar{d}) \to \pi^\circ(u\bar{u}) + \pi^+(u\bar{d})$ [figure b)].

$K^0(d\bar{s}) \to \pi^-(d\bar{u}) + \pi^+(u\bar{d})$ [figure c)]. $K^0(d\bar{s}) \to \pi^-(d\bar{u}) + \pi^+(u\bar{d})$ [figure c)].

$\Lambda(uds) \to p(udu) + e^- + \bar{\nu}_e$ [figure d)].

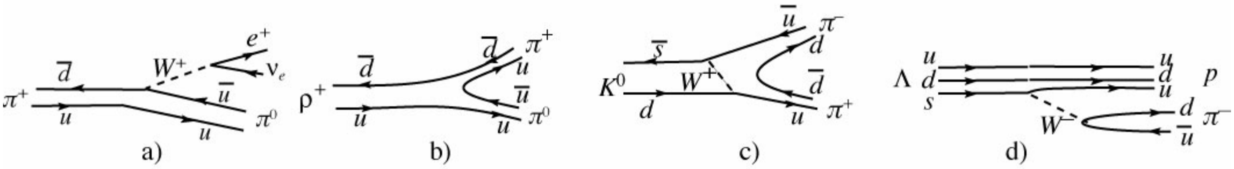

Fig. S.6

7.4. $L = 630\ \mu$m.

7.5. $\Gamma(\tau \to e\nu_e\bar{\nu}_\tau)/\Gamma(\mu \to e\nu_e\bar{\nu}_\mu) = m_\tau^5 / m_\mu^5 = 1.33 \times 10^6$,

and $\tau_\tau = \dfrac{2.2 \times 10^{-6} \times 0.16}{1.35 \times 10^6} = 2.6 \times 10^{-13}$ s .

7.6. Lepton universality gives $\sigma(e^+e^- \to \tau^+\tau^-) = \sigma(e^+e^- \to \mu^+\mu^-) = \dfrac{86.8 \text{ nb}}{s(\text{GeV}^2)}$, i. e. 0.87 nb at 10 GeV and 8.7 pb at 100 GeV.

7.8. a) $\nu_\mu + p \to \mu^- + p + \pi^+$; b) $\nu_\mu + n \to \mu^- + n + \pi^+$ and $\nu_\mu + n \to \mu^- + p + \pi^0$.

Both $\mu^+ \to e^+ + \gamma$ and $\mu^+ \to e^+ + e^+ + e^-$ violate lepton and muon flavour. They do not exist.

7.9. The quantity $\mathbf{p}_\Lambda \cdot \boldsymbol{\sigma}_\Lambda$ is a pseudoscalar. It must be zero if parity is conserved, therefore the polarisation must be perpendicular to $\mathbf{p}_\Lambda$.

7.10. We work in the CM frame. We can use two kinematic quantities, the muon momentum p that is a vector, and the muon polarisation $\boldsymbol{\sigma}$ that is an axial vector. The quantity $\mathbf{p} \cdot \boldsymbol{\sigma}$ is a pseudoscalar. If we find that the polarisation is not perpendicular to the momentum, parity is violated.

7.12. The minimum momentum of the electron is zero, with the two neutrinos emitted with the same and opposite momenta. The maximum electron momentum is when the two neutrinos have equal momenta, opposite to the electron momentum. Let us think at the process as a decay into two bodies, the electron and the two-neutrino system. The mass of the latter is $m_{2\nu}$=0 and, recalling problem 1.11, we have $p_e = \frac{m_\mu^2 - m_e^2}{2m_\mu} = 53 \text{ MeV}$.

7.13. The neutrino flux through a generic normal surface S is $\Phi=N_\nu/S$. The corresponding target is a cylinder of section S and length $2R$. Its mass is $M=\rho S2R$, containing $N_b=MN_A10^3=\rho S2RN_A10^3$ nucleons. Therefore the number of interactions is

$R = \Phi N_b \sigma = N_\nu \rho 2RN_A 10^3 \sigma = 25.2$

7.14. $\sigma\left(\nu_\mu e^- \to \nu_\mu e^-\right) \approx \frac{G_F^2}{\pi} 2m_e E_\nu$ and $\sigma\left(\nu_\mu N \to \mu^- h\right) \approx 0.2 \times \frac{G_F^2}{\pi} 2m_p E_\nu$. Therefore we have $\sigma\left(\nu_\mu N \to \mu^- h\right) / \sigma\left(\nu_\mu e^- \to \nu_\mu e^-\right) \approx 0.2 m_p / m_e = 400$.

We must now be careful with the measurement units. We have

$$\frac{\sigma\left(\nu_\mu e^- \to \nu_\mu e^-\right)}{E_\nu} \approx \frac{G_F^2}{\pi} 2m_e = \frac{\left(1.17 \times 10^{-5} \text{ GeV}^{-2}\right)^2 \times 2 \times 0.5 \text{ GeV}}{3.14} = 0.43 \times 10^{-13} \text{ GeV}^{-3},$$

which we write as

$0.43 \times 10^{-13} \text{ GeV}^{-2} / \text{GeV} = \left(0.43 \times 10^{-13} \; \mu\text{b}\right) / \text{GeV} = 0.017 \text{ fb} / \text{GeV}$.

Finally we have $\sigma\left(\nu_\mu N \to \mu^- h\right) / E_\nu \approx 400 \times 0.017 \text{ fb} / \text{GeV} = 6.8 \text{ fb} / \text{GeV}$.

7.16. In order to have a rate R=1/84600, we need $N_{71} = \frac{R}{\Phi \times \sigma \times \varepsilon} = 10^{29}$ ^{71}Ga nuclei, corresponding to $N_{\text{moli}} = N_{71} / N_A = 1.7 \times 10^5$ moles. The ^{71}Ga mass is $M_{71} = N_{\text{moli}} \times 10^{-3} \times 71 \text{ kg} = 12 \text{ t}$ and the total Ga mass is $M = M_{71} / a = 30 \text{ t}$.

7.17. The electron numerical density in iron is $n_e = \frac{Z}{A} \rho N_A 10^3 \approx 2.2 \times 10^{30} \text{ m}^{-3}$. Therefore the average distance between collisions is $L = \frac{1}{n_e \sigma} = 2.7 \times 10^{14} \text{ m}$. The corresponding time is about 10^6 s. For comparison, 1 A.U.≈1.5×10^{11} m, hence $L \approx 1800$ A.U.

7.18. The decay $c \to d + e^+ + \nu_e$ is disfavoured because its amplitude is proportional to $\sin\theta_C$. The decay $c \to s + e^+ + \nu_e$ is favoured because its amplitude is proportional to $\cos\theta_C$. We write down the valence quark compositions: $D^+ = c\bar{d}$, $K^+ = u\bar{s}$, $K^- = s\bar{u}$, $\bar{K}^0 = s\bar{d}$. Consequently the decays of D^+ in final states containing a K^- or $\bar{K}^0$ are favourite. For example, $D^+ \to K^- + \pi^+ + e^+ + \nu_e$, $D^+ \to \bar{K}^0 + e^+ + \nu_e$, $D^+ \to \bar{K}^{*0} + e^+ + \nu_e$ are favoured. $D^+ \to \pi^- + \pi^+ + e^+ + \nu_e$, $D^+ \to \pi^0 + \pi^+$ and $D^+ \to \rho^0 + e^+ + \nu_e$ are disfavoured.

7.19. $B^+ = u\bar{b}$; $\bar{D}^0 = u\bar{c}$; $D^- = d\bar{c}$.

Three favourite decays are $B^+ \to \bar{D}^0 + \pi^+$; $B^+ \to D^- + \pi^+ + \pi^+$; $B^+ \to \bar{D}^0 + e^+ + \nu_e$.

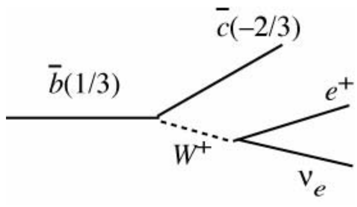

Fig. S.7

7.20. Being V_{tb} very near to 1, the dominant decay is $t \to b\ W$. There are seven diagrams

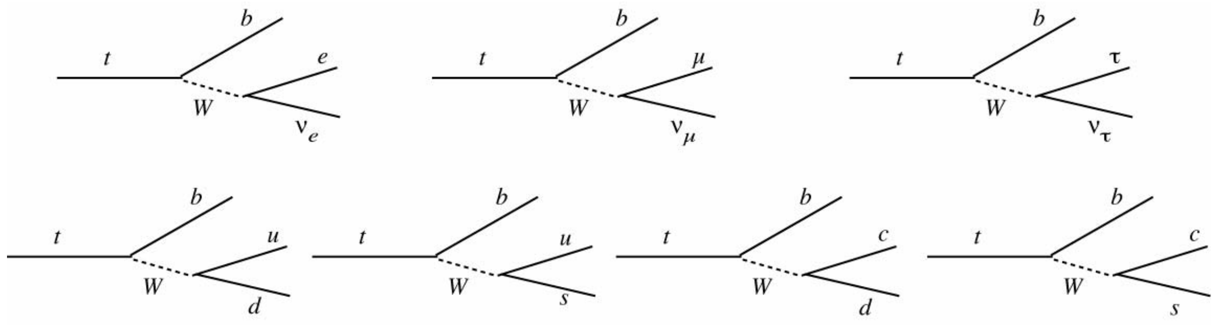

Fig. S.8

7.21. We count the different decay channels. We need only to consider the type of particle. The three semileptonic decays $b \to c + l + \nu_l$ have the same rate. The sum of the rates of $b \to c + u + d$ and $b \to c + u + s$ is again the same, multiplied by three because of the colours. The same is true for the sum of $b \to c + c + d$ and $b \to c + c + s$.

Summing up we have $\dfrac{\Gamma\left(b \to c + e^- + \bar{\nu}_e\right)}{\Gamma\left(b \to c\right)} = \dfrac{1}{1+1+1+3+3} \approx 0.1$.

7.22. We approximate the semileptonic decay of the D^+ hadron with the decay of the quark $c \to se^+\nu_e$. Then we have $\Gamma\left(c \to s + e^+ + \nu_e\right) / \Gamma\left(\mu^+ \to e^+\nu_e\bar{\nu}_\mu\right) = \left(m_c / m_\mu\right)^5 \cos^2\theta_C = 2.1 \times 10^5$, with an uncertainty of at least 35% due to the 7% uncertainty on m_c. The experimental value is 1.6×10^5.

7.23. We start by writing the valence quark contents of the hadrons, we then identify the decay at the quark level and which quark acts as a spectator.

1. At the hadron level we have $c\bar{d} \to s\bar{d} + u\bar{d}$ and at the quark level $c \to su\bar{d}$ with a spectator $\bar{d}$. The decay rate is proportional to $|V_{cs}|^2 |V_{ud}|^2 \simeq \cos^4\theta_C$.
2. At hadron level it is $c\bar{d} \to u\bar{s} + s\bar{d}$ and at the quark level it is $c \to su\bar{s}$ with a spectator $\bar{d}$. The decay probability is proportional to $|V_{cs}|^2 |V_{us}|^2 \simeq \sin^2\theta_C \cos^2\theta_C$;
3. The π^0 has a $u\bar{u}$ and a $d\bar{d}$ component. The decay picks up the latter. At hadron level it is $c\bar{d} \to u\bar{s} + d\bar{d}$ and at the quark level it is $c \to du\bar{s}$ with a spectator $\bar{d}$. The decay probability is proportional to $|V_{cd}|^2 |V_{us}|^2 \simeq \sin^4\theta_C$.

7.24. Let us consider the valence quarks of the hadrons. The decay $\Sigma^-(dds) \to n(ddu) + e^- + \bar{\nu}_e$ corresponds to $s \to u + e^- + \bar{\nu}_e$ at the quark level, with two *d*s as 'spectators'. The decay $\Sigma^+(uus) \to n(ddu) + e^+ + \nu_e$ does not correspond to $u \to d + e^+ + \nu_e$

because an initial s should transform into a final d. It is a violation of the $\Delta S=\Delta Q$ rule.

7.25. 1. $\bar{b}d \to \bar{c}d + u\bar{d} \Rightarrow \bar{b} \to \bar{c}u\bar{d}$; the decay rate is proportional to $|V_{cb}|^2 |V_{ud}|^2$;

2. $\bar{b}d \to \bar{c}d + u\bar{s} \Rightarrow \bar{b} \to \bar{c}u\bar{s}$; the decay rate is proportional to $|V_{cb}|^2 |V_{us}|^2$;

3. $\bar{b}d \to \bar{u}d + u\bar{s} \Rightarrow \bar{b} \to \bar{u}u\bar{s}$; the decay rate is proportional to $|V_{ub}|^2 |V_{us}|^2$;

4. $\bar{b}d \to \bar{u}d + u\bar{d} \Rightarrow \bar{b} \to \bar{u}u\bar{d}$; the decay rate is proportional to $|V_{ub}|^2 |V_{ud}|^2$.

The decreasing order is 1, 2, 4 and 3.

7.26. In the rest frame of the pion $p_\mu^* = p_\nu^* = \dfrac{m_\pi^2 - m_\mu^2}{2m_\pi} = 30\text{ MeV}$ and $E_\mu^* = 110\text{ MeV}$. The Lorentz factors to the laboratory frame are γ=3.84 and β=0.97. From the Lorentz transformation $p_\mu \cos\theta = \gamma\, p_\mu^* \cos\theta^* + \gamma\beta E_\mu^*$; we see that the minimum and maximum μ momentum are for θ^*=0 and θ^*=π. We have $p_{\mu,\max} = \gamma\left(p_\mu^* + \beta E_\mu^*\right) = 525\text{ MeV}$ and $p_{\mu\min} = \gamma\left(-p_\mu^* + \beta E_\mu^*\right) = 295\text{ MeV}$.

The neutrino is a left ν_μ.

7.27. The number of target electrons necessary for the interaction rate R is $N_e = \dfrac{R}{\Phi\times\sigma\times 0.5} = \dfrac{10/84600}{10^{10}\times 10^{-47}\times 0.5} = 2.4\times 10^{33}$. The number of electrons in the water mass M is $N_e = \dfrac{10}{18} M N_A 10^3$. Therefore we need M=7200 t.

7.28. There are $n_{\text{Fe}} = \rho\times N_A 10^3 / A = 10^{17}\times 6\times 10^{23}\times 10^3/56 = 1.1\times 10^{42}\text{ m}^{-3}$ nucleons per unit volume. Consequently the mean free path is $\lambda_\nu = \dfrac{1}{n_{Fe}\sigma} = \dfrac{1}{1.1\times 10^{42}\times 3\times 10^{-46}} = 3\text{ km}$.

This distance is smaller than the radius of the supernova core.

7.29.

1. $\bar{\nu}_\mu + p \to e^+ + n$.

2. At threshold $s = (m_e + m_n)^2 = (E_\nu + m_p)^2 - p_\nu^2 = m_p^2 - 2m_e E_\nu$, giving

$$E_\nu = \frac{(m_e + m_n)^2 - m_p^2}{2m_p} = \frac{(0.51 + 939.6)^2 - 938.3^2}{2\times 938.3} = 1.8\text{ MeV}.$$

3. The muon decay is $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ can be considered as the "two-body" decay $\mu^+ \to (e^+\nu_e) + \bar{\nu}_\mu$ and we have in the m^+ rest frame, $E_{\bar{\nu}_\mu} = \dfrac{m_\mu^2 + m_{\bar{\nu}_\mu}^2 - m_{e\nu_e}^2}{2m_\mu} = \dfrac{m_\mu^2 - m_{e\nu_e}^2}{2m_\mu}$, which is maximum when the $(e^+\nu_e)$ mass is minimum, namely when $m_{e\nu_e} = m_e + m_{\nu_e} = m_e$. Hence

$$E_{\bar{\nu}_\mu}^{\max} = \frac{m_\mu^2 - m_e^2}{2m_\mu} \simeq \frac{m_\mu}{2} = 53\text{ MeV}.$$

7.30. $\Sigma_c^{++}(uuc) \to \Sigma^+(uus)\pi^+;\ \ \Sigma^+(uus) \to p(uud)\pi^0$.

$\Xi_c^+(usc) \to \Xi^0(uss)\pi^+$; $\Xi^0(uss) \to \Lambda(uds)\pi^0$; $\Lambda(uds) \to p(udu)\pi^-$.

Notice that $\Xi^0(uss) \to \Sigma^+(uus)\pi^-$ is forbidden by energy conservation.

$\Omega_c^0(ssc) \to \Omega^-(sss)\pi^+$; $\Omega^-(sss) \to \Xi^0(uss)\pi^-$; $\Xi^0(uss) \to \Lambda(uds)\pi^0$; $\Lambda(uds) \to p(udu)\pi^-$

7.31. $\Sigma_b^+(uub) \to \Sigma_c^{++}(uuc)\pi^-$; $\Sigma_c^{++}(uuc) \to \Sigma^+(uus)\pi^+$; $\Sigma^+(uus) \to p(uud)\pi^0$.

$\Xi_b^-(dsb) \to \Xi_c^0(dsc)\pi^-$; $\Xi_c^0(dsc) \to \Xi^-(dss)\pi^+$; $\Xi^-(dss) \to \Lambda(dus)\pi^-$; $\Lambda(dus) \to p(udu)\pi^-$

$\Lambda_b^0(udb) \to \Lambda_c^+(udc)\pi^0$; $\Lambda_c^+(udc) \to \Lambda^0(uds)\pi^+$; $\Lambda^0(uds) \to n(udd)\pi^0$.

7.32. The decay $\Sigma^- \to \Lambda + e^- + \nu_e$ is Cabibbo favoured (rate proportional to $\cos^2\theta_C$) while $\Sigma^- \to n + e^- + \nu_e$ is Cabibbo suppressed (rate proportional to $\sin^2\theta_C$) , however the Q-value is only 82 MeV in the first case and 257 in the second and the phase space volume is consequently much larger.

8.1. From isospin conservation we have $|\phi\rangle = |0,0\rangle = \frac{1}{\sqrt{2}}|K^+\rangle|K^-\rangle - \frac{1}{\sqrt{2}}|K^0\rangle|\bar{K}^0\rangle$.

Therefore $\phi \to \bar{K}^0 K^0 / \phi \to K^+K^- = 1$. The ratio of the phase space volumes is $p_\pm^* / p_0^* = 127/110 = 1.15$.

The two mesons are in P wave, i. e. in a spatially antisymmetric state. Consequently they cannot be identical. The only possibility is $K_1^0 K_2^0$.

8.2. We start by preparing a table with the quantum numbers of the initial states

	$^1S_0$	$^3S_1$	$^1P_0$	$^3P_0$	$^3P_1$	$^3P_2$
P^{PC}	0^{-+}	1^{--}	1^{+-}	0^{++}	1^{++}	2^{++}

The two-meson final state can have J^{PC}=0^{++}, 1^{--}, 2^{++}, leaving as possible initial states $^3S_1$, $^3P_0$, and $^3P_2$. The state $K_1^0 K_1^0$ contains two identical bosons, which must have even orbital momentum, leaving $^3P_0$, and $^3P_2$ as possible initial states. The orthogonal state $K_1^0 K_2^0$ must come from $^3S_1$.

8.3. The reaction $\pi^- + p \to K^0 + \Lambda$ produces a pure K^0 state. Therefore the ratio between K_1^0 and K_2^0 is initially one.

The Lorentz factors are $\gamma = E/m \approx p/m = 20$ and $\beta \approx 1$, hence

$$I_S(l) = I_0 \exp\left(-\frac{t}{\gamma\tau_S}\right) = I_0 \exp\left(-\frac{l}{\beta c\gamma\tau_S}\right);\quad I_L(l) = I_0 \exp\left(-\frac{t}{\gamma\tau_L}\right) = I_0 \exp\left(-\frac{l}{\beta c\gamma\tau_L}\right).$$

$$\frac{I_s(l)}{I_L(l)} = \exp\left[-\frac{l}{\beta c\gamma}\left(\frac{1}{\tau_S} - \frac{1}{\tau_L}\right)\right] = \exp(-18.7) = 7.6 \times 10^{-9} .$$

8.4. $\gamma = E/m_K = 4.05$ and $\beta = 0.97$; $t = \gamma\tau \ln 10 = 1.1 \times 10^{-7}$ s and $d = \beta ct = 32$ m .

8.5. In the reaction $\pi^- p \to K^0 + X$, X must have $\mathcal{B}$=1, Q=0, S=–1. It is a hyperon. The minimum mass one is the Λ. In the reaction $\pi^- p \to \bar{K}^0 + Y$, Y must have $\mathcal{B}$=1, Q=0, S=+1. The minimum mass system is $K^0 n$. The two threshold energies are $E_\pi = \frac{(m_{K^0} + m_\Lambda)^2 - m_p^2 - m_\pi^2}{2m_p} = 910$ MeV and $E_\pi = \frac{(2m_{K^0} + m_n)^2 - m_p^2 - m_\pi^2}{2m_p} = 1520$ MeV .

8.6. For the K^0 component the channel with the lowest threshold is $K^0 p \to K^0 p\pi^0$. Its threshold energy is $E_K = \frac{\left(m_{K^0} + m_p + m_{\pi^0}\right)^2 - m_{K^0}^2 - m_p^2}{2m_p} = 714 \text{ MeV}$, corresponding to $p_{K^0} = 511 \text{ MeV}$. Only the elastic channel is therefore open at $p_{K^0} = 400 \text{ MeV}$.

The initial state $\bar{K}^0$ has strangeness S=–1 in the final state we can have a hyperon. The masses of all the systems $\pi^0\Lambda$, $\pi^0\Sigma^0$, $\pi^+\Sigma^-$ and $\pi^-\Sigma^+$ are smaller than $m_{K^0} + m_p$. Therefore these channels are open even at zero initial momentum. Consequently the $\bar{K}^0 p$ cross section is much larger than that of K^0p.

8.7. The first B lived $n = \frac{l}{\beta\gamma\tau_B c} = \frac{120\ \mu\text{m}}{257\ \mu\text{m}} = 0.47$ lifetimes, the second one 1.9 lifetimes. The μ^+ comes from a $\bar{b}$ decay (as opposite to a b decay) therefore the hadron is a B^0. The sister B has negative beauty at production but can have any beauty at decay, due to the oscillation. Therefore the second μ can have both signs.

8.8. The time needed is $\left(10^{41}\ \text{cm}^{-2}\right)/\left(10^{34}\text{cm}^{-2}\text{s}^{-1}\right) = 10^7\ \text{s}$. However, the real running time is much longer.

The initial isospin is I=0, B-mesons have I=1/2. Consequently, the square of the Clebsch Gordan coefficient for the $B^0\bar{B}^0$ final state is 1/3. The effective $B^0\bar{B}^0$ production cross section is $\sigma\left(B^0\bar{B}^0\right) = \frac{1}{3}\frac{87 \text{ nb}}{s}\Delta R = \frac{1}{3}\frac{87 \text{ nb}}{(10.6)^2}3 = 0.77 \text{ nb}$. The number of $B^0\bar{B}^0$ pairs is 77×10^6.

The average separation between production and decay is $\Delta z \approx 260\ \mu$m.

8.9. We invert the system of equation (8.32), i. e. $\sqrt{2}\left|K_L^0\right\rangle = (1+\varepsilon)\left|K^0\right\rangle + (1-\varepsilon)\left|\bar{K}^0\right\rangle$ and the corresponding one for K_S^0, i. e. $\sqrt{2}\left|K_S^0\right\rangle = (1+\varepsilon)\left|K^0\right\rangle - (1-\varepsilon)\left|\bar{K}^0\right\rangle$, taking into account that $|\varepsilon|$ is small.

We obtain $\sqrt{2}\left|K^0\right\rangle = (1-\varepsilon)\left|K_S^0\right\rangle + (1-\varepsilon)\left|K_L^0\right\rangle$, $\sqrt{2}\left|\bar{K}^0\right\rangle = (1+\varepsilon)\left|K_S^0\right\rangle - (1+\varepsilon)\left|K_L^0\right\rangle$. The decay amplitudes can be written as

$$\sqrt{2}A\left(\bar{K}^0 \to \pi^+\pi^-\right) = (1+\varepsilon)\left[A\left(K_S^0 \to \pi^+\pi^-\right) - A\left(K_L^0 \to \pi^+\pi^-\right)\right] = A\left(K_S^0 \to \pi^+\pi^-\right)(1+\varepsilon)(1-\eta_{+-})$$

and similarly $\sqrt{2}A\left(K^0 \to \pi^+\pi^-\right) = A\left(K_S^0 \to \pi^+\pi^-\right)(1-\varepsilon)(1+\eta_{+-})$. We finally obtain

$$\left|\frac{A\left(\bar{K}^0 \to \pi^+\pi^-\right)}{A\left(K^0 \to \pi^+\pi^-\right)}\right| = \left|\frac{(1+\varepsilon)-(1+\varepsilon)\eta_{+-}}{(1-\varepsilon)+(1-\varepsilon)\eta_{+-}}\right| = \left|\frac{(1+\varepsilon)-(1+\varepsilon)(\varepsilon+\varepsilon')}{(1-\varepsilon)+(1-\varepsilon)(\varepsilon+\varepsilon')}\right| \approx 1 - 2\,\text{Re}\,\varepsilon'$$

8.10. Following solution **8.9** we have

$$\left|\frac{A\left(\bar{K}^0 \to \pi^0\pi^0\right)}{A\left(K^0 \to \pi^0\pi^0\right)}\right| = \left|\frac{(1+\varepsilon)-(1+\varepsilon)(\varepsilon-2\varepsilon')}{(1-\varepsilon)+(1-\varepsilon)(\varepsilon-2\varepsilon')}\right| \approx 1 + 4\,\text{Re}\,\varepsilon'.$$

8.11. $\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = \frac{\left|1-2\frac{\varepsilon'}{\varepsilon}\right|^2}{\left|1+\frac{\varepsilon'}{\varepsilon}\right|^2} = \frac{\left(1-2\,\mathrm{Re}\frac{\varepsilon'}{\varepsilon}\right)^2 + 4\left(\mathrm{Im}\frac{\varepsilon'}{\varepsilon}\right)^2}{\left(1+\mathrm{Re}\frac{\varepsilon'}{\varepsilon}\right)^2 + \left(\mathrm{Im}\frac{\varepsilon'}{\varepsilon}\right)^2} \simeq \frac{1-4\,\mathrm{Re}\frac{\varepsilon'}{\varepsilon}}{1+2\,\mathrm{Re}\frac{\varepsilon'}{\varepsilon}}$. And finally

$\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 \simeq \left(1-4\,\mathrm{Re}\frac{\varepsilon'}{\varepsilon}\right)\left(1-2\,\mathrm{Re}\frac{\varepsilon'}{\varepsilon}\right) \simeq 1-6\,\mathrm{Re}\frac{\varepsilon'}{\varepsilon}$.

8.12. If *CP* is conserved, in the decay $K_2^0 \to \pi^+\pi^-\pi^0$ *L*=*l*=0. The spatial wave-function of the dipion is even. The total must be even. Consequently, the isospin wave function of the dipion must be even, hence $I_{\pi\pi}$ =0 or $I_{\pi\pi}$ =2.

In the decay $K_1^0 \to \pi^+\pi^-\pi^0$, *L*=*l*=odd. The isospin wave function of the dipion must be odd, hence $I_{\pi\pi}$ =1 or $I_{\pi\pi}$ =3.

9.1. The requested interaction rate is *R*=1/(4×Δ*t*)=42/s. On the other hand we have $R=\Phi\sigma N_e$

The number of electrons is 1/2 of the number of nucleons. Therefore their number in the fiducial mass *M* is $N_e = \frac{1}{2}MN_A 10^3 = 1.64\times 10^{32}$. The cross section at E_ν=24 GeV is σ=4.1×10^{-44} m^2 = 0.41 fb. The requested flux is $\Phi = R/\left(\sigma N_e\right) = 1.5\times 10^{13}\ \mathrm{s^{-1}m^{-2}}$ and the beam intensity $I = \Phi\times A = 1.6\times 10^{14}\ \mathrm{s^{-1}}$. The duty cycle is $2\Delta t/T = 0.8\times 10^{-3}$.

9.2. $\theta_e \le \sqrt{2m_e/E_e} = 7\ \mathrm{mrad}$. The accuracy should be at least $\sigma_\theta \approx 2-3\ \mathrm{mrad}$. The calorimeter must be built with low *Z* materials to have a small multiple scattering, not with Fe.

9.3. In the rest frame of the pion the neutrino energy is $E_\nu^* = \left(m_\pi^2 - m_\mu^2\right)/\left(2m_\pi\right) = 30\ \mathrm{MeV}$.

The Lorentz factors are $\gamma = E_\pi/m_\pi = 1429$ and $1-\beta = 1-\sqrt{1-\gamma^{-2}} \approx \frac{1}{2}\gamma^{-2} = 2.4\times 10^{-7}$.

The neutrino energy in the L frame is $E_\nu = \gamma\left(E_\nu^* + \beta p^*\cos\theta^*\right) = \gamma E_\nu^*\left(1+\beta\cos\theta^*\right)$. Its maximum, for θ*=0 is $E_\nu^{\max} = \gamma E_\nu^*\left(1+1\right) = 1429\times 30\times 10^{-3}\ (\mathrm{GeV})\ 2 = 85.7\ \mathrm{GeV}$. Its minimum for θ*=π is $E_\nu^{\min} = \gamma E_\nu^*\left(1-\beta\right) = 10\ \mathrm{keV}$.

We use the Lorentz transformations of the components of the neutrino momentum to find the relationship between the angle θ^* in CM and θ in L.

$p_\nu \sin\theta = p^*\sin\theta^*;\ p_\nu\cos\theta = \gamma\left(p^*\cos\theta^* + \beta E_\nu^*\right) \simeq \gamma p^*\left(\cos\theta^*+1\right)$, which gives

$$\tan\theta = \frac{\sin\theta^*}{\gamma\left(\cos\theta^*+1\right)} \simeq \frac{0.05}{1429\times 2} = 22\times 10^{-6} \Rightarrow \theta = 22\mu\mathrm{rad}\,.$$

9.4. The quark level main process for neutrinos is $\nu_{\mu L} + d_L \to \mu_L^- + u_L$ and for antineutrinos is $\bar{\nu}_{\mu R} + u_L \to \mu_R^+ + d_L$. The kinematics are the same as those in Fig. 9.5. In the first case the angular momentum is *J*=0. In the second case it is *J*=1, with third component (on the incident direction) J_z=+1, which is one of the possible three. This gives the factor 3.

9.7. For each reaction we check whether charge *Q* and hypercharge *Y* are conserved. We write explicitly the hypercharge values.

For $W^- \to d_L + \bar{u}_L$ we have 0–>1/3–4/3. It violates *Y*.

For $W^- \to u_L + \bar{u}_L$ we have $0 \to 1/3 - 4/3$. It violates Y and Q.
For $Z \to W^- + W^+$ we have $0 \to 0+0$. OK. For $W^+ \to e_R^+ + \bar{\nu}_R$, $0 \to 1+1$, violates Y.
9.8. $d_L \to W^- + u_L$ OK. $Z \to e_L^+ + e_L^-$ violates Y. $W^+ \to Z + W^+$ OK. $W^+ \to e_L^- + \bar{\nu}_L$ does not exist.

9.9. $\Gamma_\nu = \frac{G_F M_Z^3}{3\sqrt{2}\pi}\left(\frac{1}{2}\right)^2 \simeq 660 \times 1/4 \text{ MeV} = 165 \text{ MeV}$.

$$\Gamma_l = \frac{G_F M_Z^3}{3\sqrt{2}\pi}\left[\left(-\frac{1}{2}+s^2\right)^2 + s^4\right] \simeq 660 \times 0.148 \simeq 98 \text{ MeV}.$$

$$\Gamma_u = \Gamma_c = 3\frac{G_F M_Z^3}{3\sqrt{2}\pi}\left[\left(\frac{1}{2}-\frac{2}{3}s^2\right)^2 + \left(-\frac{2}{3}s^2\right)^2\right] \simeq 3 \times 660 \times 0.173 \simeq 342 \text{ MeV}.$$

$$\Gamma_d = \Gamma_s = \Gamma_b = 3\frac{G_F M_Z^3}{3\sqrt{2}\pi}\left[\left(-\frac{1}{2}+\frac{1}{3}s^2\right)^2 + \left(\frac{1}{3}s^2\right)^2\right] \simeq 3 \times 660 \times 0.207 \simeq 410 \text{ MeV}.$$

$\Gamma_Z = 3 \times 165 + 3 \times 98 + 2 \times 342 + 3 \times 410 = 2.7 \text{ GeV}$.

$\Gamma_h = 2 \times 342 + 3 \times 410 = 1910 \text{ MeV}$; $\Gamma_\mu / \Gamma_h = \frac{98}{1910} = 5.1\%$.

9.10. $\Gamma_\nu = 165 \text{ MeV}$, $\Gamma_e \simeq 83 \text{ MeV}$, $\Gamma_u = \Gamma_c \simeq 275 \text{ MeV}$, $\Gamma_d = \Gamma_s = \Gamma_b \simeq 360 \text{ MeV}$, $\Gamma_Z = 2374 \text{ MeV}$, $\Gamma_h = 1630 \text{ MeV}$; $\Gamma_\mu / \Gamma_h = 5.1\%$.

9.11. The W couples universally to the leptons and to the quarks of each colour. However, it cannot decay into $t\bar{b}$ because its ass is too small. Therefore the open channels are $W^+ \to e^+ + \nu_e$; $W^+ \to \mu^+ + \nu_\mu$; $W^+ \to \tau^+ + \nu_\tau$; $3 \times W^+ \to u + \bar{d}'$; $3 \times W^+ \to c + \bar{s}'$. Neglecting the masses of the final fermions we evaluate $\frac{W^+ \to e^+\nu_e}{\text{total}} = \frac{1}{1+1+1+3+3} = 0.11$.

9.12. $\frac{g_{Zee}^2}{g_W^2} = \frac{\left(-1/2+s^2\right)^2 + s^4}{1/2} = 0.25$ and $\frac{\Gamma\left(Z \to e^+e^-\right)}{\Gamma\left(W \to e^+\nu_e\right)} = \frac{g_{Zee}^2}{g_W^2}\frac{M_Z^3}{M_W^3} = 0.25 \times 1.45 = 0.36$.

9.13. $\frac{g_{Zuu}^2}{g_W^2} = \frac{3\left[\left(1/2 - 2s^2/3\right)^2 + \left(-2s^2/3\right)^2\right]}{3 \times 1/2} = 0.29$ and $\frac{\Gamma\left(Z \to \bar{u}u\right)}{\Gamma\left(W \to \bar{u}d\right)} = \frac{g_{Zuu}^2}{g_W^2}\frac{M_Z^3}{M_W^3} = 0.41$.

9.14. $\sigma\left(\bar{u}d \to \bar{q}q\right) = \sigma\left(\bar{u}d \to \bar{u}d'\right) + \sigma\left(\bar{u}d \to \bar{c}s'\right) = 6 \times \sigma\left(\bar{u}d \to e^+\nu_e\right) = 60 \text{ nb}$.

9.15.
$\Gamma_Z = 3\Gamma_l + 2\Gamma_u + 3\Gamma_d + N\Gamma_\nu = 3 \times 83 + 2 \times 280 + 3 \times 360 + N \times 166 = 1889 + N \times 166$.

$\frac{\Gamma_\mu}{\Gamma_Z}(3) = 3.48\%$, $\frac{\Gamma_\mu}{\Gamma_Z}(4) = 3.25\%$ and $\frac{\Gamma_\mu}{\Gamma_Z}(5) = 3.05\%$.

$\sigma_0^h(3) : \sigma_0^h(4) : \sigma_0^h(5) = \Gamma_Z^{-2}(3) : \Gamma_Z^{-2}(4) : \Gamma_Z^{-2}(5) = 1 : 0.87 : 0.77$.

9.16. $\sigma\left(e^+ + e^- \to \mu^+ + \mu^-\right) = \frac{12\pi}{m_Z^2}\frac{\Gamma_e\Gamma_\mu}{\Gamma^2} = 5.4 \times 10^{-6} \text{ GeV}^{-2} \times 389\ \mu\text{b/GeV}^{-2} = 2.1 \text{ nb}$.

$\sigma\left(u\bar{u} \to q\bar{q}\right) = \frac{4\pi}{3}\frac{1}{M_Z^2}\frac{\Gamma_u\Gamma_h}{\Gamma_Z^2} = 4 \times 10^{-5} \times 389\ \mu\text{b} = 16 \text{ nb}$,

$$\sigma\left(u\bar{d} \to e^+\bar{\nu}_e\right) = \frac{4\pi}{3}\frac{1}{M_W^2}\frac{\Gamma_{\bar{u}d}\Gamma_{e\nu}}{\Gamma_W^2} = 2.3\times 10^{-5} \times 389\ \mu\text{b} = 9\ \text{nb}\ .$$

9.17. The energy of a Z of momentum $p_Z = 140$ GeV is $E_Z = 167$ GeV and the Lorentz parameters are $\gamma_Z = p_Z / M_Z = 1.54$ and $\beta_Z = p_Z / E_Z = 0.84$. In the CM frame the components of the momenta of the electrons perpendicular to the beams are $p_n^{*+}=-p_n^{*-}$=45 GeV, while their longitudinal components are zero. It is also $E^{*+}=E^{*-}$=45 GeV. In the L frame, $p_n^{+}=p_n^{*+}$=45 GeV, $p_n^{-}=p_n^{*-}$=–45 GeV. The longitudinal momentum and the energy of both the electron and the positron are $p_L = 0 + \gamma_Z\beta_Z E^* = 58$ GeV and $E_L = \sqrt{p_L^2 + p_n^2 + m_Z^2} = 117$ GeV. Their angles at the two sides of the beams are $\theta_L = \tan^{-1}\left(p_n / p_L\right) = 38°$.

9.18. $m^2 = 4E_1E_2\sin^2\theta/2 \quad \Rightarrow m = 92$ GeV .

$$\frac{\sigma_{M_Z}}{M_Z} = \frac{1}{2}\sqrt{\left(\frac{\sigma(E_1)}{E_1}\right)^2 + \left(\frac{\sigma(E_2)}{E_2}\right)^2 + \left(\frac{\sigma(\theta)}{\tan\theta/2}\right)^2} = \frac{1}{2}10^{-2}\sqrt{2.4^2 + 2^2 + 0.6^2} = 1.6\%\ .$$

9.19. $\dfrac{\delta M_W}{M_W} = \dfrac{\delta\sin\theta_W}{\sin\theta_W} = \dfrac{1}{2}\dfrac{\delta\sin^2\theta_W}{\sin^2\theta_W}$, therefore $M_W = 80 \pm 8$ GeV .

$$\frac{\delta\sin^2\theta_W}{\sin^2\theta_W} = \frac{\delta\cos^2\theta_W}{\cos^2\theta_W} = 2\frac{\delta\left(m_W / m_Z\right)}{m_W / m_Z} = 2\%\ .$$

9.20. The weak interaction strength is proportional to g^2/M_W^2.

9.21. The energy squared in the quark-antiquark CM frame is $\hat{s} = x_q x_{\bar{q}} s$. Assuming, for the sake of our evaluation, $x_q = x_{\bar{q}}$, we have $x_q = x_{\bar{q}} = \sqrt{\hat{s}/s} = M_Z / \sqrt{s} = 0.045$. The sea quarks structure functions are about $x\bar{d}(0.045) \approx x\bar{u}(0.045) \approx xd(0.045) \approx 0.5xu(0.045)$.

The momentum fraction of the Z with longitudinal momentum P_Z=100 GeV is $x_Z = x_q - x_{\bar{q}} = p_Z / p_{\text{beam}} = 0.1$. By substitution into $m_Z^2 = x_q x_{\bar{q}} s$ we obtain $m_Z^2 = x_q\left(x_q - 0.1\right)s$ and $x_q^2 - 0.1x_q - \dfrac{m_Z^2}{s} = 0$ or, numerically, $x_q^2 - 0.1x_q - 0.002 = 0$.

Its solution is $x_q = 0.1 \pm \sqrt{0.1^2 + 4\times 0.002} = 0.234$. The other solution is negative and therefore not physical.

9.22. $R=\mathcal{L}\sigma=1.7\times 10^{-4}\ \text{s}^{-1}$, i. e. about 14 events a day.

9.23. $\Gamma_\nu / \Gamma_Z = 6.6\%$. From $\sigma_0 = \dfrac{12\pi}{M_Z^2}\dfrac{\Gamma_h\Gamma_e}{\Gamma_Z^2}$ we have $\Delta\sigma_0 / \sigma_0 = 2\,\Delta\Gamma_Z / \Gamma_Z = 13\%$.

9.24. The measurable quantity that is most sensitive to the invisible width is the hadronic cross section at the peak. We have already calculated in problem 9.15 that $\sigma_0^h(3):\sigma_0^h(4) = 1:0.87$. For a five standard deviation effect we need a statistical uncertainty of 0.13/5=2.6%. The corresponding number of events is given by $\sqrt{N}/N = 0.026$, about 1500 events.

9.25. R for 3, 4 and 5 neutrinos is 10.0, 10.7 and 11.4. Therefore R<10.1 would exclude more than 3 neutrinos. Notice however that the result depends somewhat on the exact values of the widths of Z and W.

9.26. The target contains the same number of up and down quarks. In the charged currents case neutrinos interact as $\nu_{\mu L} + d_L \to \mu_L^- + u_L$, antineutrinos as $\bar{\nu}_{\mu R} + u_L \to \mu_R^+ + d_L$. As we saw in problem 9.4, in the latter case (*LR*) there is a factor 1/3 relative to the former (*LL*). Therefore it is $\sigma_{CC}(\nu_L)/\sigma_{CC}(\bar{\nu}_R) = 3$.

All the target quarks u_R, u_L, d_R and d_L contribute to the neutral currents interactions each proportionally to its *Z*-charge factor squared c_Z^2. We sum their contributions taking into account the 1/3 factor for the *LR* and *RL* contributions relative to the *LL* and *RR* contributions.

We have $\dfrac{\sigma_{NC}(\nu_L)}{\sigma_{NC}(\bar{\nu}_R)} = \dfrac{\left[c_Z^2(u_L) + c_Z^2(d_L)\right] + \frac{1}{3}\left[c_Z^2(u_R) + c_Z^2(d_R)\right]}{\frac{1}{3}\left[c_Z^2(u_L) + c_Z^2(d_L)\right] + \left[c_Z^2(u_R) + c_Z^2(d_R)\right]}$, giving

$$\frac{\sigma_{NC}(\nu)}{\sigma_{NC}(\bar{\nu})} = \frac{\left[\left(\frac{1}{2} - \frac{2}{3}s^2\right)^2 + \left(-\frac{1}{2} + \frac{1}{3}s^2\right)^2\right] + \frac{1}{3}\left[\left(-\frac{2}{3}s^2\right)^2 + \left(\frac{1}{3}s^2\right)^2\right]}{\frac{1}{3}\left[\left(\frac{1}{2} - \frac{2}{3}s^2\right)^2 + \left(-\frac{1}{2} + \frac{1}{3}s^2\right)^2\right] + \left[\left(-\frac{2}{3}s^2\right)^2 + \left(\frac{1}{3}s^2\right)^2\right]} = \frac{\frac{1}{2} - s^2 + \frac{20}{27}s^4}{\frac{1}{3}\left(\frac{1}{2} - s^2 + \frac{20}{9}s^4\right)} = 2.3 .$$

9.27. We are in the rest frame of an object of mass 2 m_χ 'decaying' into two equal bodies. If these are two photons we have $m_\chi = E_\gamma^* = 136\text{ GeV}$. If the final particles are a photon and a *Z* we have $E_\gamma^* = \dfrac{4m_\chi^2 - m_Z^2}{2m_\chi}$, giving $m_\chi^2 - E_\gamma^* m_\chi - \dfrac{m_Z^2}{4} = 0$. Solving this equation and discarding the unphysical negative solution, we obtain $m_\chi = \dfrac{1}{2}\left(E_\gamma^* \pm \sqrt{E_\gamma^{*2} + m_Z^2}\right) = 150\ \text{ GeV}$.

9.28. 1. For the symmetric configuration the colour force is repulsive, while it is attractive for the antisymmetric configuration.

2 and 3. Both violate the hypercharge.

9.29. 1. a. OK; b. violates *Y*; c. conserves *Y*, but violates colour; d. OK; e. violates *Y*; f. conserves *Y*, but is forbidden by the change of flavour; g. violates energy.

2. The answer is no.

9.30. At resonance $s = M_W^2 = \left(E_\nu + m_e\right)^2 - p_\nu^2 = m_e^2 + 2m_e E_\nu \simeq 2m_e E_\nu$

The resonance neutrino energy is $E_\nu = \dfrac{M_W^2}{2m_e} = 6.4\text{ PeV}$, which is too high for any accelerator.

9.31. We can use the approximations of problem 1.22 b). The maximum momentum transfer is $p_{\max} = 2p_{\nu,\max} = 2E_{\nu,\max} = 32\text{ MeV}$. Not enough to resolve the structure, which require a momentum transfer larger than $\dfrac{197\text{ MeV fm}}{R_A} = 50\text{ fm}$.

The maximum recoil kinetic energy (non relativistic) is

$$E_{k,r} = \frac{p_{\max}^2}{2M_{Ge}} = \frac{2E_{\nu,\max}^2}{M_{Ge}} = \frac{2 \times 16^2}{76 \times 10^3} = 6.7\text{ keV}$$

Being a neutral current interaction the cross section is flavour independent.

9.32. The Z-charges squared of the u and d quarks are:

$$c_Z^2(u_L) = \left(\frac{1}{2} - \frac{2}{3}s^2\right)^2 = 0.12\,, \quad c_Z^2(d_L) = \left(-\frac{1}{2} + \frac{1}{3}s^2\right)^2 = 0.18\,, \quad c_Z^2(u_R) = \left(-\frac{2}{3}s^2\right)^2 = 0.024\,,$$

$$c_Z^2(d_R) = \left(\frac{1}{3}s^2\right)^2 = 0.006\,.$$

The neutrino cross section on an u quark is proportional to

$$c_Z^2(u_L) + \frac{1}{3}c_Z^2(u_R) = \frac{1}{4} - \frac{2}{3}s^2 + \frac{16}{27}s^4 = 0.25 - 0.15 + 0.03 = 0.13$$

and that on a d quark to

$$c_Z^2(d_L) + \frac{1}{3}c_Z^2(d_R) = \frac{1}{4} - \frac{1}{3}s^2 + \frac{4}{27}s^4 = 0.25 - 0.08 + 0.008 = 0.18\,.$$

The cross section on a nucleus containing the same number of u and d quarks (only) is proportional to

$$c_Z^2(u_L) + c_Z^2(d_L) + \frac{1}{3}\left[c_Z^2(u_R) + c_Z^2(d_R)\right] = \frac{1}{2} - s^2 + \frac{20}{27}s^4 = 0.50 - 0.23 + 0.04 = 0.31\,.$$

9.33. The Z-charges squared of the u and d quarks have been calculated in problem 9.3. The difference now is that antineutrinos have positive helicity, then the factor 1/3 is for the L quarks.

$$c_Z^2(u_L) = \left(\frac{1}{2} - \frac{2}{3}s^2\right)^2 = 0.12\,, \quad c_Z^2(d_L) = \left(-\frac{1}{2} + \frac{1}{3}s^2\right)^2 = 0.18\,, \quad c_Z^2(u_R) = \left(-\frac{2}{3}s^2\right)^2 = 0.024\,,$$

$$c_Z^2(d_R) = \left(\frac{1}{3}s^2\right)^2 = 0.006$$

The neutrino cross section on an u quark is proportional to

$$\frac{1}{3}c_Z^2(u_L) + c_Z^2(u_R) = \frac{1}{3}0.12 + 0.024 = 0.064\,.$$

and that on a d quark to: $\frac{1}{3}c_Z^2(d_L) + c_Z^2(d_R) = \frac{1}{3}0.18 + 0.006 = 0.066\,.$

The cross section on a nucleus containing the same number of u and d quarks (only) is proportional to $\frac{1}{3}\left[c_Z^2(u_L) + c_Z^2(d_L)\right] + c_Z^2(u_R) + c_Z^2(d_R) = 0.013\,.$

9.34. NC couplings are the same for different families. The ratio is 1.

9.35.

e, V: $c_Z(e_L) + c_Z(e_R) = -\frac{1}{2} + 2s^2 = -0.50 + 046 = 0.04$

e, A: $c_Z(e_L) - c_Z(e_R) = -\frac{1}{2} = -0.50$

u, V: $c_Z(u_L) + c_Z(u_R) = \frac{1}{2} - \frac{4}{3}s^2 = 0.50 - 031 = 0.19$

u, A: $c_Z(u_L) - c_Z(u_R) = \frac{1}{2} = 0.50$

d, V: $c_Z(d_L) + c_Z(d_R) = -\frac{1}{2} + \frac{2}{3}s^2 = -0.50 + 015 = 0.35$

d, A: $c_Z(d_L) - c_Z(d_R) = -\frac{1}{2} = -0.50$

ν_e, being neutrino only left, both A and V are $\frac{1}{2}$

9.36.

a) We start from the results of problem 9.6. The protons contains 2 u quarks and one d quark. Hence its axial Z-charge is $c_{ZV}(p) = 2c_{ZV}(u) + c_{ZV}(d) = 1 - \frac{8}{3}s^2 - \frac{1}{2} + \frac{2}{3}s^2 = \frac{1}{2} - 2s^2 \simeq 0.04$.

And that of the neutron is $c_{ZV}(n) = c_{ZV}(u) + 2c_{ZV}(d) = \frac{1}{2} - \frac{4}{3}s^2 - 1 + \frac{4}{3}s^2 = \frac{1}{2} = 0.5$.

b) The axial Z-charge of a nucleus with Z protons and N neutrons is $\frac{1}{2}\left[N + \left(1 - 4s^2\right)Z\right]$ and is dominated by the neutron contribution.

c) The vector Z-charge of the electron is very small (0.04) compared to its axial charge (–0.50)

10.1. The sensitive mass of M=100 t of C_9H_{12} of molecular mass $M_{C_9H_{12}}$ contains

$N_{C_9H_{12}} = \frac{M}{M_{C_9H_{12}}} N_A = \frac{10^8}{120} 6 \times 10^{23} = 5 \times 10^{29}$ molecules and $N_{el} = N_{C_9H_{12}} \times 66 = 3.3 \times 10^{31}$ electrons.

1. Number of events in one day if no oscillation

$$N_{nosc} = N_{el}\sigma(\nu_e e)\Phi_\nu \times 86400(\text{s/d}) = 3.3 \times 10^{31} \times 0.6 \times 10^{-48} \times 4.6 \times 10^{13} \times 8.64 \times 10^4 = 77.$$

2. The mechanism is vacuum oscillation

3. The survival probability well below resonance is

$$P_{ee} \simeq 1 - \frac{1}{2}\sin^2 2\theta_{12} = 1 - \frac{1}{2}\sin^2 68° = 0.57$$

Expect 77×0.57=44 from $\nu_e e$ scattering and

$77 \times (1 - 0.57) \times \frac{\sigma(\nu_{\mu,\tau} e)}{\sigma(\nu_e e)} = 77 \times 0.43 \times \frac{1}{6} = 6$ events from ν_μ and ν_τ scatterings.

In total 50 events/(100 t day)

10.2.

1. Muon neutrinos

2. In the π CM frame energy and momentum of the neutrinos are

$$p_\nu^* = E_\nu^* = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} = \frac{139.6^2 - 105.7^2}{2 \times 139.6} = 29.8 \text{ MeV}$$

The Lorentz factor is $\gamma = \frac{E_\pi}{m_\pi} = \frac{5}{0.1396} = 35.8$

The Lorentz transformations

$p_\nu^* \sin\theta_\nu^* = p_\nu \sin\theta_\nu \simeq p_\nu \theta_\nu$

$p_\nu^* \cos\theta_\nu^* = \gamma\, p_\nu \left(1 - \beta \cos\theta_\nu\right) \simeq \gamma\, p_\nu \frac{\theta_\nu^2}{2}$

Hence $\tan\theta_\nu^* = \frac{2}{\gamma\theta_\nu} = \frac{2}{35.8 \times 0.044} = 1.27 \qquad \theta_\nu^* = 52^\circ$

$p_\nu = p_\nu^* \frac{\sin\theta_\nu^*}{\sin\theta_\nu} = 540 \text{ MeV}$.

3. For $\theta_\nu = 0$ it is also $\theta_\nu^* = 0$, and $p_\nu^* = \gamma\, p_\nu (1-\beta)$. Hence: $p_\nu = \frac{p_\nu^*}{\gamma(1-\beta)}$.

$$\gamma^2 = \frac{1}{1-\beta^2} = \frac{1}{(1-\beta)(1+\beta)} \simeq \frac{1}{2(1-\beta)} \qquad \Rightarrow \qquad \gamma(1-\beta) = \frac{1}{2\gamma}$$

and $p_\nu = 2\gamma\, p_\nu^* = 2130$ MeV

The number of nucleons in M=22.5 t of water is

$N_N = M \times 10^3 \times N_A = 2.25 \times 10^{10} \times 6 \times 10^{23} = 1.35 \times 10^{34}$.

The number of CC ν_μ interactions in absence of oscillations would be $N_i = N_N \Phi\sigma = 1.35 \times 10^{34} \times 2 \times 10^{11} \times 3 \times 10^{-43} = 800$.

The disappearance probability is

$$P\left(\nu_\mu \to \nu_x\right) = \sin^2 2\theta_{23} \cos^2\theta_{13} \sin^2\left(1.27 \Delta m^2 \frac{L}{E_\nu}\right) \simeq \sin^2\left(1.27 \times 2.5 \times 10^{-3} \frac{295}{0.54}\right) =$$

$$= \sin^2(1.73) = 0.97$$

1. The ν_e appearance probability is

$$P\left(\nu_\mu \to \nu_e\right) = \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2\left(1.27 \Delta m^2 \frac{L}{E_\nu}\right) \simeq 2\theta_{13}^2 \sin^2\left(1.27 \times 2.5 \times 10^{-3} \frac{295}{0.54}\right) =$$

$$= 1.5 \times 10^{-2} \sin^2(1.73) = 1.5 \times 10^{-2}$$

10.3. The kinematic of the problem has been solved in §6.2. The relevant equation, written in the variables of the present problem is $E_\nu m_N = E_\mu E_\nu + E_\mu m_N - p_\mu E_\nu \cos\theta_\mu - m_\mu^2$, which gives

$$E_\nu = \frac{E_\mu m_N - m_\mu^2}{m_N - E_\mu + p_\mu \cos\theta_\mu}$$

and numerically $E_\nu = \frac{0.5 \times 0.938 - 0.106^2}{0.938 - 0.5 + \sqrt{0.5^2 - 0.106^2}\cos 30^\circ} = 0.53$ GeV

10.4.

1. The number of oxygen nuclei is $N_{^{16}O} = \frac{M \times 10^3 \times N_A}{18} = \frac{22.5 \times 10^9 \times 6 \times 10^{23}}{18} = 7.5 \times 10^{32}$

and the number of interactions in one year is

$N_{\nu_\mu O} = N_{^{16}O} \Phi_\nu \Delta\Omega\sigma = 7.5 \times 10^{32} \times 130 \times 1 \times 3 \times 10^7 \times 10^{-42} = 30$

3. The survival probability is 50%

4. The length of the chord at 90° is $L = 2R\sin\frac{90^\circ}{2} = 2 \times 6.378 \times 10^3 \times 0.77 = 9.0 \times 10^3$ km

The survival probability is

$$1 - P\left(\nu_\mu \to \nu_x\right) = 1 - \sin^2 2\theta_{23} \sin^2\left(1.27\Delta m^2 \frac{L}{E_\nu}\right) \simeq 1 - 1 \times \sin^2\left(1.27 \times 2.5 \times 10^{-3} \frac{9 \times 10^3}{1}\right) =$$

$$= 1 - \sin^2\left(29^\circ\right) = 0.76$$

10.5. The number of water molecules is $N_{H_2O} = \frac{M \times 10^3 \times N_A}{18} = \frac{22.5 \times 10^9 \times 6 \times 10^{23}}{18} = 7.5 \times 10^{32}$ and the number of electrons $N_{el} = 10 \times N_{H_2O} = 7.5 \times 10^{33}$. The number of interactions in one year is

$$N_{\nu_e ES} = N_{H_2O} \Phi\sigma\varepsilon \times 3 \times 10^7\,(\text{s/yr}) = 7.5 \times 10^{33} \times 10^{10} \times 10^{-47} \times 0.5 \times 3 \times 10^7 = 11000 .$$

The measured rate is about ½ due to flavour conversion in the Sun via MSW effect.

10.6. a) CNGS. The Lorentz factor is $\gamma = \frac{E_\pi}{m_\pi} = \frac{80}{0.1396} = 573$

and the decay length $l_\pi = c\gamma\tau = 3 \times 10^8 \times 573 \times 1.6 \times 10^{-8} = 2.75$ km .

T2K. $\gamma = \frac{E_\pi}{m_\pi} = \frac{7}{0.1396} = 50$, $l_\pi = 240$ m

(b) The CM momentum, which is also the neutrino energy in the pion decay, is

$$p^* = E_\nu^* = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} = 29.8 \text{ MeV} .$$

Consider a neutrino emitted at the angle θ* to the beam in the CM frame and let us transform to the L frame

$$p_\nu \sin\theta = p_\nu^* \sin\theta^*$$

$$p_\nu = E_\nu = \gamma\left(E_\nu^* + \beta p_\nu^* \cos\theta^*\right) = \gamma p_\nu^*\left(1 + \beta\cos\theta^*\right)$$

and we have

$$p_{\nu,\max} = \gamma\left(1 + \beta\right) p_\nu^* \simeq 2\gamma p_\nu^* \text{, for } \theta^* = 0$$

$$p_{\nu,\min} = \gamma\left(1 - \beta\right) p_\nu^* \simeq \frac{1}{2\gamma} p_\nu^*$$

CNGS: $p_{\nu,\max} = 33$ GeV; $p_{\nu,\min} = 25$ keV

T2K: $p_{\nu,\max} = 2.9$ GeV; $p_{\nu,\min} = 300$ keV

(c) The momentum components of a neutrino at θ^*=90° are

Transverse component $p_{\nu y} = p_\nu^*$; Longitudinal component $p_{\nu x} = \gamma p_\nu^*$

The angle in the L frame is $\theta \simeq \tan\theta \simeq \frac{1}{\gamma}$

CNGS $\theta = 0.9$ mrad . Beam "radius" @ OPERA $R = 7.3 \times 10^5 \times 1.4 \times 10^{-3} \simeq 0.6$ km

T2K $\theta = 20$ mrad . Beam "radius" @ SuperK $R = 2.95 \times 10^5 \times 2 \times 10^{-2} \simeq 5.9$ km

10.7.

a) The threshold energy is

$$E_\nu = \frac{\left(m_\tau + m_p\right)^2 - m_n^2}{2m_n} = \frac{\left(1777 + 938.3\right)^2 - 939.6^2}{2 \times 939.6} = 3.45 \text{ GeV}.$$

b) The number of target nucleons is $N_t = M \times N_A \times 10^3 = 2 \times 10^{12} \times 6 \times 10^{23} = 1.2 \times 10^{36}$.

The number of CC interactions per year is

$N_{\text{int}} = N_\nu \sigma_{CC} N_t = 4.3 \times 10^8 \times 10^{-41} \times 1.2 \times 10^{36} = 5160$

c) The oscillation probability into ν_τ is

$$P\left(\nu_\mu \to \nu_\tau\right) = \sin^2\left(2\theta_{23}\right)\cos^4\left(\theta_{13}\right)\sin^2\left(1.27\Delta m^2 \frac{L}{E}\right) \simeq 1 \times \sin^2\left(1.27 \times 2.5 \times 10^{-3} \times \frac{730}{18}\right) = 0.017$$

and the number of ν_τ per year (same cross section) is 87.

d) The oscillation probability into ν_e is

$$P\left(\nu_\mu \to \nu_e\right) = \sin^2\left(\theta_{23}\right)\sin^2\left(2\theta_{13}\right)\sin^2\left(1.27\Delta m^2 \frac{L}{E}\right) \simeq 2\theta_{13}^2 \times 0.017 = 5 \times 10^{-4}$$

and the expected number of ν_e interactions per year would be 2.6.

10.8. a) Pair production and Compton scattering

b) As in problem 6.5, the maximum transfer is for background scattering, namely θ=180, is

$$E_{k,\max} = \frac{2E_\nu^2}{m_p} = \frac{4 \times 3^2}{938} = 38 \text{ keV}.$$

c) Calling E_+ the energy of the positron, energy conservation in the process $\bar{\nu}_e + p \to e^+ + n$ gives $E_\nu = E_+ + m_n - m_p$, were we have neglected the recoil energy that we have seen to be very small. Energy conservation in $e^+ + e^- \to 2\gamma$ gives $E_{vis} = E_+ + m_e$. In conclusion

$E_\nu = E_{vis} + m_n - m_p - m_e = E_{vis} + 0.8 \text{ MeV}$.

d) Since $E_{vis} \geq 2m_e \approx 1 \text{ MeV}$, the detected neutrinos have energy $E_\nu \geq 1.8 \text{ MeV}$.

10.9. The area at the distance L_1 is $A_1 = 4\pi L_1^2 = 1.3 \times 10^5 \text{ m}^2$ and the neutrino flux $\Phi_1 = \frac{N}{A_1} = \frac{6 \times 10^{20}}{1.3 \times 10^5} = 4.5 \times 10^{15}\text{/s} = 4 \times 10^{20}\text{/d}$. The interaction rate is $R = \Phi N_p \sigma$. Hence the necessary number of target protons is $N_p = \frac{10^2}{\Phi_1 \sigma} = \frac{10^2}{4 \times 10^{20} \times 10^{-47}} = 25 \times 10^{27}$, that is 40 000 mole or M_1=40 kg.

At L_2 that is 20 times larger the flux is 400 times smaller and the necessary proton mass is 16 t.

10.10. The number of interaction in one year on N_t target protons is $N_{\text{int}} = \Phi_\nu P_{ee} \sigma N_t$, hence the number of free protons needed is

$$N_t = \frac{N_{\text{int}}}{3.1 \times 10^7 \left(\text{s/yr}\right) \times \Phi_\nu \times f \times P_{ee} \times \sigma} = \frac{10^3}{3.1 \times 10^7 \times 3.5 \times 10^{10} \times 0.05 \times 0.6 \times 10^{-47}} = 3.1 \times 10^{33}$$

An effective mole of the blend contains

$N_{free} = \left(0.20 \times 18 + 0.80 \times 26\right) N_A = 24.4 \times 6 \times 10^{23} = 1.46 \times 10^{25}$ protons/mol. Hence we need

$\frac{N_t}{N_{free}} = 2.1 \times 10^8$ effective mol.

The weighted molar mass is $M_A = 0.20 \times 210 + 0.80 \times 218 = 266$ g. The blend mass needed is

$$M = \frac{N_t}{N_{free}} M_A \times 10^3 = 2.1 \times 10^8 \times 0.266 = 56 \times 10^6 \text{ kg}$$

10.11. The number of nucleon per unit volume is $n_n = \rho \times 10^3 \times N_A = 1.5 \times 10^{30}$ and the mean free path is $L = \frac{1}{n_n \sigma_{CC}} = \frac{1}{2.8 \times 10^{-36} \times 1.5 \times 10^{30}} = 240 \text{ km}$. Earth is quite opaque to these neutrinos.

10.12. ^{76}Ge. Number of nuclei in one ton $N_{Ge} = \frac{10^3 \times 10^3 \times N_A}{76} = \frac{10^6 \times 6 \times 10^{23}}{76} = 8 \times 10^{27}$. One half of them decay in a $T_{1/2}$. Hence 4 decays per year

^{136}Te. Number of nuclei in one ton $N_{Te}=4.6\times10^{27}$. Hence 2.3 decays per year.

^{130}Xe. Number of nuclei in one ton $N_{Xe}=4.4\times10^{27}$. Hence 2.2 decays per year.

10.13. (a) The kinetic energy $E_{k,B} \ll m_\nu$. Neutrinos are non-relativistic. $\beta = \sqrt{\frac{2E_{k,B}}{m_\nu}} = 0.07$. Their energy is close to the mass energy $E_{\nu B} \simeq m_\nu = 100$ meV

(b) From $s \simeq 2E_\nu E_{\nu B}$, we have $E_\nu = \frac{m_Z^2}{2E_{\nu B}} = \frac{91^2}{2 \times 10^{-10}} = 4.1 \times 10^{13}$ GeV

(c) We have $\sigma(\nu_x \bar{\nu}_x \to \nu_x \bar{\nu}_x) = \left(\frac{\Gamma_\nu}{\Gamma_l}\right)^2 \sigma(e^+e^- \to \mu^+\mu^-) = 1.99^2 \times 2.1 \text{ nb} = 8.4 \text{ nb}$. The mean free path is then $\lambda = \frac{1}{\sigma(\nu_x \bar{\nu}_x \to \nu_x \bar{\nu}_x)\rho} = \frac{1}{8.4 \times 10^{-37} \times 5.6 \times 10^7} = 2 \times 10^{28}$ m, which is larger than the radius of the universe.